No Spurious Welfare Gains from Taxation:
A Further Argument for the Equivalent Variation*

Julio R. Robledo† & Andreas Wagener‡

Abstract

Among all measures based on money metric compensation functions, solely the excess burden based on the equivalent variation always correctly identifies the excess burden of a specific tax to be positive. This provides a new argument for the use of the equivalent variation when measuring the welfare effects of a price change.

Keywords: Welfare Measures, Excess Burden, Equivalent Variation.

JEL Classification: D61, H2.

*We are grateful to Egbert Dierker and Hildegard Dierker for helpful comments.
†Department of Economics, University of Vienna, Hohenstaufengasse 9, 1010 Vienna, Austria. Phone/fax: +43 1 4277 37429 / 9374. E-mail: julio.robledo@univie.ac.at.
‡Corresponding author. Institute of Social Policy, University of Hannover, Koenigsworther Platz 1, 30167 Hannover, Germany. Phone/fax: +49 511 762 5874 / 4574. E-mail: wagener@sopo.uni-hannover.de.
1 Introduction

Monetary values for changes in individual well-being that are caused by price variations are, in both theoretical and practical analyses, often measured in terms of changes in Marshallian consumer’s surplus or by Hicksian equivalent or compensating variations. The change in Marshallian consumer’s surplus is path dependent when there are multiple price changes (Dixit and Weller, 1979). The compensating variation fails to rank two or more welfare changes consistently (Hause, 1975; Chipman and Moore, 1980). The equivalent variation does not suffer from either of these shortcomings. Ebert (1995) axiomatically characterizes the equivalent variation as the only money-metric measure whose sign correctly reflects the direction of welfare change and that correctly ranks two or more price changes with respect to a common status quo. Moreover, in public finance, minimizing the excess burden of the tax system calculated on the basis of the equivalent variation is equivalent to the Ramsey problem of setting distortionary taxes in a welfare-maximizing way (Auerbach and Hines, 2002).

We add a further argument for the use of the equivalent variation, taken from the field of taxation. Distortionary taxes harm individuals in excess of taking away their money. Indicators for this excess burden of taxation can be constructed as differences between the money-valued change in consumer’s welfare (however measured) and actual tax revenues. As taxation imposes an extra cost on taxpayers beyond the revenues collected, a reasonable measure of the excess burden should only take positive values. Many measures of the excess burden violate this property. As discussed in Mayshar (1990), the excess burden based on the compensating variation may be negative if the tax is imposed on an inferior good. Also, the change in Marshallian consumer’s surplus falsely identifies a negative deadweight loss if the taxed item is a Giffen good (Facchini et al., 2001). This note extends these results. For the class of excess burden measures based on compensation functions, it shows that misleadingly identifying a negative excess burden of distortionary taxation can be avoided if and only if the excess burden is measured on the basis of the equivalent variation.

2 The framework

A single consumer maximizes utility by choosing over consumption bundles \( x = (x_1, x_2) \) comprised of two consumption goods 1 and 2; the restriction to two goods is innocuous. The consumer has an exogenous lump-sum income \( y > 0 \) and takes commodity prices
(p_1, p_2) as given. His utility function u(x) is assumed to be smooth and strictly quasi-concave. We use good 2 as the (always untaxed) numéraire and notationally suppress p_2 \equiv 1 henceforth. We denote by x_1(p_1, y) and x_2(p_1, y) the consumer’s Marshallian demand functions and by v(p_1, y) his indirect utility function. The dual program of expenditure minimization yields Hicksian (compensated) demand functions h_1(p_1, u) and h_2(p_1, u) and expenditure function e(p_1, u).

In the initial situation, both goods are untaxed. Due to a specific excise tax with rate t_1 > 0, the price of good 1 increases from, say, p_1^A to p_1^A + t_1. To assess the welfare effects of such a price change, the literature has focused on three measures (Dixit and Weller, 1979). For an arbitrary reference price \( \bar{p}_1 \in \mathbb{R}^+ \), the compensation function 
\[
M(\bar{p}_1; p_1^A, t_1, y) := e(\bar{p}_1, v(p_1^A + t_1, y)) - e(\bar{p}_1, v(p_1^A, y))
\] (1)
defines a money-valued measure of the welfare change relative to the reference price \( \bar{p}_1 \).

Putting \( \bar{p}_1 = p_1^B \) in (1) yields the compensating variation \( CV(t_1) := M(p_1^A + t_1; p_1^A, t_1, y) \), while putting \( \bar{p}_1 = p_1^A \) gives the equivalent variation \( EV(t_1) := M(p_1^A; p_1^A, t_1, y) \). The change in Marshallian consumer’s surplus is \( \Delta CS(t_1) := -\int_{p_1^A}^{p_1^A + t_1} x_1(p_1, y) dp_1 \). Although these three measures and, more generally, measures based on different compensation functions typically differ in magnitude, they are equal in sign: For all \( t_1 > 0 \), they are negative, indicating a loss in individual well-being due to the tax-induced price increase.

3 Negative excess burdens of a tax

Tax revenues from levying a specific excise tax at rate \( t_1 \) on good 1 are \( T(t_1) = t_1 \cdot x_1(p_1^A + t_1, y) \), but this amount generally differs from the monetary valuation of the consumer’s utility loss. As shown by Facchini et al. (2001), the excess burden measured on the basis of the change in consumer’s surplus \( EB^{CS}(t_1) := -\Delta CS(t_1) - T(t_1) \) is negative whenever the taxed item is a Giffen good, i.e., if \( \partial x_1(p_1, y) / \partial p_1 > 0 \) for all \( p_1 \in [p_1^A, p_1^A + t_1] \).

Exhibiting such “spurious” welfare gains is certainly a disturbing feature – but one that \( EB^{CS} \) shares with the excess burden measured on the basis of the compensating variation, \( EB^{CV}(t_1) := -CV(t_1) - T(t_1) \). Mayshar (1990) has already remarked that \( EB^{CV}(t_1) \) may be negative if an inferior good is taxed. The following result shows that tax revenues in fact always exceed the compensating variation if an excise tax is levied on a Giffen good:

**Proposition 1** If \( \partial x_1(p_1, y) / \partial p_1 > 0 \) for all \( p_1 \in [p_1^A, p_1^A + t_1] \), then \( EB^{CV}(t_1) < 0 \).
Proof: By definition, $EB^{CV}(0) = 0$. Now differentiate $EB^{CV}(t_1)$ with respect to $t_1$:

$$
\frac{\partial EB^{CV}(t_1)}{\partial t_1} = h_1(p^A_1 + t_1, v(p^A_1, y)) - x_1(p^A_1 + t_1, y) - t_1 \cdot \frac{\partial x_1(p^A_1 + t_1, y)}{\partial p_1}
$$

$$
= [x_1(p^A_1 + t_1, y - CV(t_1)) - x_1(p^A_1 + t_1, y)] - t_1 \cdot \frac{\partial x_1(p^A_1 + t_1, y)}{\partial p_1}
$$

$$
< 0.
$$

The second equality follows from the implicit definition of $CV(t_1), v(p^A_1, y) = v(p^A_1 + t_1, y - CV(t_1))$, and the duality properties of Hicksian and Marshallian demand. If good 1 is a Giffen good, then $\partial x_1/\partial p_1$ is positive. As Giffen goods are inferior, the square-bracketed expression above is negative due to $CV < 0$. Hence, $EB^{CV}(t_1) < 0$ for all $t_1 > 0$. ■

Based on $EB^{CV}$, the taxation of a Giffen good with subsequent reimbursement of tax revenues seems to result in a utility improvement. Even more oddly, this improvement would be greater the higher the tax rate (since $\partial EB^{CV}/\partial t_1 < 0$). The source of this defect of $EB^{CV}$ differs, however, from that of the namely defect of $EB^{CS}$: For the latter it is omitted income effects, while for $EB^{CV}$ it is an “improper” reference point – as we shall see now.

4 Correctly identifying an excess burden

For a given reference price $\tilde{p}_1$ and compensation function $M(\tilde{p}_1; p^A_1, t_1, y)$, the excess burden of a tax at rate $t_1$ can be measured by the difference between the negative of $M$ and the tax revenues raised:¹

$$
EB(\tilde{p}_1; p^A_1, t_1, y) := -M(\tilde{p}_1; p^A_1, t_1, y) - T(t_1)
$$

$$
= e(\tilde{p}_1, v(p^A_1, y)) - e(\tilde{p}_1, v(p^A_1 + t_1, y)) - t_1 \cdot x_1(p^A_1 + t_1, y). \quad (2)
$$

The next result shows that $EB(\tilde{p}_1; p^A_1, t_1, y)$ reliably measures the excess burden, i.e., that it always identifies that the compensation an individual requires in lieu of taxation exceeds the delivered tax revenues, if and only it is computed on the basis of the equivalent variation.

Proposition 2 For all $(p^A_1, y) \in \mathbb{R}^2_+, EB(\tilde{p}_1; p^A_1, t_1, y) > 0$ for all $t_1 \in \mathbb{R}_+$, if and only if $\tilde{p}_1 = p^A_1$.

¹Observe that this definition uses actual revenues rather than income-compensated, hypothetical tax revenues; see Mayshar (1990, Sect. 4.3) for an elaboration of this aspect.
Proof: The “if-part” is well-known; see, e.g., Mas-Colell et al. (1995, p. 84) for a proof that $EB(p_1A; p_1^A, t_1, y) > 0$ always holds.

To show the “only-if-part”, let $EB(\bar{p}_1; p_1^A, t_1, y)$ first undergo a series of transformations:

$$EB(\bar{p}_1; p_1^A, t_1, y) =$$

$$= \int_{p_1^A}^{p_1^A + t_1} \left( -\frac{\partial e(\bar{p}_1, v(p_1, y))}{\partial u} \cdot \frac{\partial v(p_1, y)}{\partial p_1} - x_1(p_1^A + t_1, y) \right) dp_1$$

$$= \int_{p_1^A}^{p_1^A + t_1} \left( \frac{\partial e(\bar{p}_1, v(p_1, y))}{\partial u} \cdot \frac{\partial v(p_1, y)}{\partial y} \cdot x_1(p_1, y) - x_1(p_1^A + t_1, y) \right) dp_1$$

$$= \int_{p_1^A}^{p_1^A + t_1} \left( \frac{\partial e(\bar{p}_1, v(p_1, y))}{\partial u} \cdot \frac{\partial v(p_1, y)}{\partial u} \cdot x_1(p_1, y) - x_1(p_1^A + t_1, y) \right) dp_1. \quad (3)$$

The first equality comes from converting the difference into an integral and the second uses Roy’s Identity. The third stems from the duality property that the Lagrangian multipliers of the utility maximization problem (i.e., the marginal utility of income) and of the expenditure minimization problem (i.e., the derivative of the expenditure function with respect to the utility level) are inverse to each other.\(^2\)

Instead of showing the “only-if-part” directly, we show its logical equivalent, namely that for $\bar{p}_1 \neq p_1^A$, there exists a $(p_1^A, t_1, y)$ such that $EB(\bar{p}_1; p_1^A, t_1, y) < 0$, e.g. such that the integrand in (3) becomes negative for all $p_1 \in (p_1^A, p_1^A + t_1)$. Then the integral itself will be negative too. This strategy of proof allows us to rely on suitable examples. The preferences in the following Cases I and II were selected merely for reasons of tractability and do not restrict the generality of our proof; similar effects could be obtained with other preferences as well.

Case I: $\bar{p}_1 < p_1^A$. Consider a Cobb-Douglas utility function $u(x_1, x_2) = x_1^\gamma \cdot x_2^{1-\gamma}$ for $0 < \gamma < 1$. Then $x_1(p_1, y) = \gamma \cdot y/p_1$. Moreover, the expenditure function is linear in $u$ with a constant coefficient $\partial e(p_1, u)/\partial u = p_1^A$. Thus, the integrand in (3) reads as:

$$\frac{\gamma \cdot y}{p_1^A} \cdot \left[ \left( \frac{\bar{p}_1}{p_1} \right)^\gamma - \frac{p_1^A}{p_1^A + t_1} \right].$$

For $\bar{p}_1 < p_1^A$ this expression will be negative for all $p_1 \in (p_1^A, p_1^A + t_1)$ whenever

$$t_1 > t_1(\bar{p}_1) := p_1^A \cdot \left[ \left( \frac{p_1^A}{\bar{p}_1} \right)^\gamma - 1 \right].$$

Suppose $\lambda^*$ is the value of the Lagrangian multiplier associated with the problem $\max\{u(x_1, x_2)|p_1 x_1 + x_2 \leq y\}$ in the optimum and $\mu^*$ is the value of the Lagrangian multiplier associated with the problem $\min\{p_1 x_1 + x_2|u(x_1, x_2) \geq \bar{u}\}$. Then $\mu^* = 1/\lambda^*$ whenever $u = v(p_1, y)$ (see, e.g., Silberberg and Suen, 2001, ch. 10). Now recall that $\lambda = \partial e/\partial y$ and $\mu = \partial e/\partial u$. 

\(^5\)
I.e., for any \( \bar{p}_1 < p_1^A \) we can find a strictly positive \( t_1(\bar{p}_1) \) such that \( EB < 0 \) whenever the tax rate \( t_1 \) exceeds \( t_1(\bar{p}_1) \).

**Case II: \( \bar{p}_1 > p_1^A \).** Consider the following utility function (this is a special case of equation (35) in LaFrance, 1985):
\[
u(x_1, x_2) = (1 - x_1) \cdot \exp\left(\frac{x_1 + x_2 - \gamma}{1 - x_1}\right)
\]
with \( \gamma > 0 \) and sufficiently large. This utility function exhibits positive marginal utilities and a decreasing marginal rate of substitution whenever \( \gamma - x_2 > x_1 > 1 \). Marshallian demand for good 1 is given by \( x_1(p_1, y) = \gamma + p_1 - y \), which makes good 1 a Giffen good. The expenditure function \( e(p_1, u) = p_1 + [\gamma - 1 + u \cdot \exp(-p_1)] \) is linear in \( u \). Hence, the integrand in (3) becomes:
\[
I(\bar{p}_1, p_1, t_1) := \exp(p_1 - \bar{p}_1) \cdot [\gamma + p_1 - y] - [\gamma + p_1^A + t_1 - y].
\]
As \( I(\cdot) \) is increasing in \( p_1 \) and as \( p_1 \in [p_1^A, p_1^A + t_1] \),
\[
I(\bar{p}_1, p_1, t_1) < [\exp(p_1^A + t_1 - \bar{p}_1) - 1] \cdot [\gamma + p_1^A + t_1 - y],
\]
which will be negative whenever \( t_1 < \bar{p}_1 - p_1^A \). Hence, whenever \( \bar{p}_1 > p_1^A \), there exist strictly positive tax rates \( t_1 \in (0, \bar{p}_1 - p_1^A) \) such that \( EB < 0 \).

In summary, whenever \( \bar{p}_1 \neq p_1^A \), then there are cases such that \( EB(\bar{p}_1; p_1^A, t_1, y) \) is negative for some configuration of prices and taxes. This completes the proof.

Rather than taking a detour via (3), we could have constructed the examples in the proof above by straightly calculating \( EB(\bar{p}_1) \). However, (3) allows us to precisely identify why and when measures other than \( EB(p_1^A) \) fail to correctly identify excess burdens. To see this, have a closer look at the fraction in the integrand in (3):
\[
\alpha(\bar{p}_1, p_1) := \frac{\partial e(\bar{p}_1, v(p_1, y))}{\partial u} / \frac{\partial e(p_1, v(p_1, y))}{\partial u}. \tag{4}
\]
Clearly \( \alpha(p_1, p_1) = 1 \). Changing the reference price \( \bar{p}_1 \) will only affect the numerator of \( \alpha \). Using Schwarz’ Rule and Shephard’s Lemma we obtain:
\[
\text{sgn} \frac{\partial \alpha}{\partial \bar{p}_1} = \text{sgn} \frac{\partial}{\partial p_1} \left( \frac{\partial e(\bar{p}_1, u)}{\partial u} \right) = \text{sgn} \frac{\partial}{\partial u} \left( \frac{\partial e(\bar{p}_1, u)}{\partial p_1} \right) = \text{sgn} \frac{\partial h_1(\bar{p}_1, u)}{\partial u} = \text{sgn} \left( \frac{\partial x_1(p_1, e(p_1, u))}{\partial y} \cdot \frac{\partial e(p_1, u)}{\partial u} \right) \tag{5}
\]
since \( h_1(p_1, u) = x_1(p_1, e(p_1, u)) \). As \( \partial e/\partial u > 0 \), (5) is positive [negative] if good 1 is normal [inferior]. Now consider the following two cases (along which the examples in the proof were constructed):

(i) Good 1 is normal. Thus, \( x_1(p_1, y) > x_1(p_1^A + t_1, y) \) for all \( p_1 \in [p_1^A, p_1^A + t_1) \). Furthermore, \( \alpha \) is increasing in \( \bar{p}_1 \). Hence, setting \( \bar{p}_1 \geq p_1^A \) yields \( \alpha(\bar{p}_1, p_1) \geq 1 \) for all \( p_1 \in [p_1^A, p_1^A + t_1) \) and therefore \( EB(\bar{p}_1; p_1^A, t_1, y) \) will always be positive. Choosing \( \bar{p}_1 < p_1^A \), however, leads to \( \alpha(\bar{p}_1, p_1) < 1 \) such that \( EB > 0 \) cannot be ensured and, as Case I in the proof of Proposition 2 demonstrates, will eventually be violated.

(ii) Good 1 is locally\(^3\) a Giffen good. Hence, \( x_1(p_1, y) < x_1(p_1^A + t_1, y) \) for all \( p_1 \in (p_1^A, p_1^A + t_1] \) and, since Giffen goods are inferior, \( \alpha \) is decreasing in \( \bar{p}_1 \). Therefore, \( \alpha(\bar{p}_1, p_1) > 1 \) for all \( p_1 \in (p_1^A, p_1^A + t_1] \) is a necessary condition for the integrand in (3) to be positive everywhere. Since \( \alpha(p_1, p_1) = 1 \) for all \( p_1 \), this can at best be ensured if \( \bar{p}_1 \leq p_1^A \). Case II in the proof of Proposition 2 confirms this.

These cases single out \( \bar{p}_1 = p_1^A \) or, equivalently, measuring the excess burden based on the equivalent variation as the only way to ensure that \( EB(\bar{p}_1; p_1^A, t_1, y) \) is positive.

5 Conclusion

We provide a further argument why one should use the equivalent variation in welfare comparisons of price changes. Unlike the axiomatic characterization by Ebert (1995), which is based on consistency requirements for assessing two or more tax changes, our argument is based on the idea that, as distortionary taxes generate deadweight losses, meaningful measures for the excess burden of taxation should reflect this by taking positive values only. This neither happens when the excess burden is based on Marshallian consumer’s surplus (due to ignoring the income effects) nor when it is based on the compensating variation or other compensation functions (due to improper reference prices). Among all measures based on compensation functions, the excess burden based on the equivalent variation is the only one that always correctly identifies the deadweight loss.

\(^3\)It is well-known that within the standard framework of utility maximization goods cannot be of the Giffen-type over the full positive range of prices and incomes.
References


