Why is the Public Sector More Labor-Intensive?

A Distortionary Tax Argument

Panu Poutvaara†
Department of Economics, University of Helsinki

Andreas Wagener‡
Department of Economics, University of Vienna

Abstract
Government-run entities are often more labor-intensive than private companies, even with identical production technologies. This need not imply slack in the public sector, but may reflect a wage tax advantage, stemming from the fact that government entities (partly) pay their taxes to themselves. A tax-induced cost advantage of public production precludes production efficiency and reduces welfare when labor supply is inelastic. With an elastic labor supply, a wage tax advantage of the public sector may improve welfare if it allows for a higher net wage.

JEL-classification: L33, J45, D24, H21.

Keywords: Public Sector, Labor Intensity, Taxation.

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†Department of Economics, University of Helsinki, Arkadiankatu 7 (P.O. Box 17), 00014 University of Helsinki, Finland. Phone: +358 9 1912 8797 Fax: +358 9 1912 8736. E-mail: panu.poutvaara@helsinki.fi.

‡Department of Economics, University of Vienna, Hohenstauffengasse 9, 1010 Vienna, Austria. Phone: (+43) 1 4277 37423. Fax: (+43) 1 4277 9374. E-mail: andreas.wagener@univie.ac.at.
1 Introduction

Around the world, the large-scale privatization and outsourcing programs of governments over the past 20 years typically went along with substantial reductions of labor input in the newly privatized firms or sectors,\(^1\) indicating that production in the public sector is often more labor-intensive than in the private sector. Quite typically, this observation is interpreted as evidence that overmanning and slack working practices prevail under government-ownership, inefficiencies that private firms could not afford under the pressure of the marketplace. Lack of competition and incentives, X-inefficiency, a soft budget constraint, and the failure to properly price inputs and outputs result in significant misallocations of resources. Moreover, political influence, infiltration by unions, or the deliberate use of employment in the public sector as a redistributive device augment the tendency towards hiring too many staff to government entities. While from everyday experience we would not dismiss the inefficiency hypothesis for the public sector entirely, we propose in this paper a different explanation why private firms have leaner workforces than state-run firms: they operate under a different tax structure with respect to factor inputs.

Consider an economy where all government expenses are financed out of a wage tax. If workers hired by a private firm in this economy are to earn a certain net wage, the employing firm faces a higher labor cost (namely, the gross wage) out of which the tax has to be transferred to the government. If the employing unit is state-owned and its recruiting staff fully sees through the government’s budgeting, then the employees’ net wage reflects the full labor cost — since, by consolidating accounts, all intra-government tax payments net out. Thus, the government will, *ceteris paribus* and optimally, hire a larger workforce than an otherwise identical private firm that produces the same level of output.

So far, this line of reasoning is based on two restrictive assumptions: constant factor

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\(^1\)For a survey see Megginson and Netter (2001). In a study on 63 privatizations, Dewenter and Malatesta (2001) report a significant decline in labor intensity after privatization. In a study on 218 privatizations in Mexico, La Porta and López-de-Silanes (1999) found that output on average increased by 54.3% while employment declined by almost half, which indicates a substantial increase in labor productivity. An opposite effect seems to be indicated by Megginson et al. (1994): for their sample of 18 countries they find that privatization on average was followed by a rise in employment. However, this effect is largely driven by changes in output or production programs after privatization and only sets in in the medium term.
prices and tax rates and a government that perfectly sees through its consolidated budget and recognizes that it pays taxes to itself. Both assumptions might be considered as inappropriate. First, an economy where one sector is government-operated differs from an otherwise identical economy where this sector is run privately with respect to factor allocations, factor prices, output in other sectors, factor supplies, budgetary needs, and tax rates. Second, the public sector is nowhere a monolithic actor but public production is typically spread over different jurisdictional levels, ministries, agencies or departments. As employers, such governmental units can only consider that fraction of the total tax burden on labor as irrelevant in terms of labor cost that actually flows back (or is perceived to accrue) to themselves.\(^2\)

Our modelling captures these two aspects. We consider the general equilibrium of a two-sector economy. One of the sectors is always privately organized (such that its labor costs correspond to gross wages) while the other sector can be either privately-run or government-controlled. The output of this sector is provided to citizens free of charge by the government, and it is financed by a wage tax that distorts labor supply. We call the economy a *private economy* if the sector that produces the government-provided good is privately run, and a *mixed economy* otherwise. In a mixed economy, the government recognizes that a fraction of the wage taxes which it pays for its employees will return to its own budget and, thus, does not constitute a genuine factor cost. For a monolithic government with a fully consolidated budget that recycling fraction would be one while with more fragmented governments, recruitment decisions will be based on fractions below one. We show that in the general equilibrium of a mixed economy the labor intensity in the government sector is higher the larger the fraction of tax revenues that returns (or is perceived to return) to the employing government units.

Observe that this result does not require that tax or social security laws actually stipulate a factual tax advantage of government employers over private employers. The differences in labor intensities are only driven by the fact that tax payments by government units (partly) net out in consolidated government budgets. However, the actual existence of preferential tax treatments for government bodies would augment the effect we are

\(^2\)There is indeed evidence that local governments are responsive to tax incentives, for example as concerns the VAT treatment of their activities. Wassenaar and Gradus (2004) compare its effect on outsourcing for seven EU countries and Norway. They find that a refund scheme for VAT costs of local governments facilitates outsourcing.

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depicting. In many countries and instances, special tax rules apply to government-run entities (often to the dismay of private competitors). In some cases (e.g., corporate income taxes, property taxes) these rules favor factors other than labor, in other cases (e.g., VAT) the effect on relative factor prices is unclear. Moreover, in some countries (e.g., in Germany, Italy, and Austria), civil servants are fully or partly exempt from social security taxes (old-age income, unemployment, or health insurance; see, e.g., Cardona, 2002). To the extent to which it relies on civil servants as its personnel, the government enjoys a labor cost advantage over the private sector which might translate into a higher labor intensity.

At first sight, a mixed economy with a public and a private sector that face different factor price ratios and, thus, produce with different marginal rates of factor substitution looks like a non-optimal arrangement relative to a fully privatized economy. However, it may be worthwhile to tolerate this violation of production efficiency. A comparison between a mixed and a private economy reveals that the equilibrium net wage may (but need not) be higher in the former. If labor supply increases with the net wage, a mixed economy will then operate with a larger labor supply than the private economy. It might, thus, produce a higher level of output than the private economy, despite the deadweight loss caused by an inefficient intersectoral allocation of resources. Hence, society may face a trade-off between production efficiency (realized in the private economy) and smaller tax distortions (best realized in the mixed economy). We identify conditions (in terms of the elasticity of labor supply) such that either welfare loss is preferable to the other.

The rest of our paper is organized as follows: Section 2 reviews related literature. Section 3 presents the model. In Section 4 we then derive the differences in factor allocations, factor prices, and tax rates that result from the different organizational modes in a mixed and in a private economy. Section 5 reports our main findings on welfare comparisons. Section 6 concludes.

2 Related Literature

An extensive literature discusses why private firms are more productive than public enterprises (see Shleifer, 1998, for a survey). However, as observed in Mintz et al. (2000), taxation is a largely overlooked issue in this debate. If taxation plays a role, then the focus is mainly on differential treatment of the public and the private sector by tax laws
and the differences in factor costs and price ratios that it implies.

Our focus is more on the organization of public production by a government which partly, if not fully, sees through its budgetary branches. This relates our paper to an earlier literature on public production and optimal taxation (see, e.g., Stiglitz and Dasgupta, 1971; Dasgupta and Stiglitz, 1972; Diamond and Mirrlees, 1971, 1976; Naito, 1999) which routinely assumes that government has a complete understanding of the economy. While in a mixed economy where the government has access to an unlimited set of tax instruments, production efficiency is desirable, some deviation from an intersectorially efficient factor allocation is typically optimal if the range of tax instruments is limited. While we reach a similar conclusion, our approach differs. Rather than presupposing a grand social planner, we ask how governmental units, be they municipalities, ministries or other agencies, behave in the labor market when they are price-takers. Moreover, instead of aiming at a second- or third-best tax structure, we take an empirically plausible, but generally non-optimal tax structure (that entails a relatively high and distortionary tax on labor) as given and analyze the best arrangement of production under this regime.

In this respect, our paper relates to a recent strand of the literature that investigates the optimal role and size of the public sector in general equilibrium models with (non-optimal) distortionary taxes. E.g., Huizinga and Nielsen (2001) investigate the optimal boundary between public and private production in a model where a range of production activities can, with different technologies, be carried out by either the government or by the private sector. Huizinga and Nielsen (2001) predict that the size of the public sector, measured by the range of activities that are carried out through the state, is larger the higher is the budgetary need for, or the marginal damage resulting from, distortionary taxation. Moreover, contracting out from the government sector generally goes along with a decrease in the use of the taxed factor. Our paper comes to quite similar conclusions

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3More precisely, if the government can set a profit tax of 100 per cent and choose all commodity taxes optimally, then public production should be organized in a way such that shadow price ratios for factors equal the marginal rates of substitution for production and consumption in the private sector of the economy.

4Huizinga and Nielsen (2001) focus on capital income taxation (which distorts private investment decisions) and predict over-capitalization of the public sector. This is at odds with reality (see also Gordon, 2003) but replacing capital by labor taxation would render the model’s forecasts compatible with reality.
— but without having to resort to differences in the efficiencies of private and public production.

Gordon et al. (1999) argue that, if the (exogenous) inefficiency of the public sector is less than proportionately related to its size while the efficiency costs of taxation increase more than proportionately with the tax rate, nationalization of industries at some point gets cheaper than financing government purchases through distortionary taxation. This result rests on an in-built inefficiency in the government sector. By contrast, the inefficiency in our model is an intersectoral one: For efficiency, sectors should not face different factor price ratios — but in a mixed economy with one sector being private and the other public they do in the presence of non-uniform factor taxation.

Inefficient public production may also be explained by redistributive motives (Alesina et al., 2000). In Pirttilä and Tuomala (2005) the government aims to transfer resources from high- to low-skilled workers. A way to achieve this without distortionary taxation is to employ more low-skilled workers and fewer high-skilled workers than cost minimization would demand. Gordon (2003) suggests that public firms may be more labor-intensive than private ones when the government hires workers that would otherwise be unemployed, or hires unskilled workers to drive up their equilibrium wage. Our model uses a representative individual such that equity concerns cannot arise. Moreover, we assume competitive and clearing factor markets, leaving no need for labor market interference by the government. Public production in our model is potentially worthwhile as it reduces the government’s financial needs, thereby allowing for lower taxes, higher net wages, and an increased labor supply.

3 The Model

3.1 Production

Consider a closed economy with two sectors \( i = 1, 2 \). Sector \( i \) uses labor \( L_i \) and capital \( K_i \) to produce its output; there are no intermediate inputs. Technologies are represented by neoclassical production functions \( F_i = F_i(L_i, K_i) \) which are assumed to have the standard monotonicity and concavity properties. Denoting partial derivatives by subscripts, we assume, in particular, that \( F_{L_i} > 0, F_{K_i} > 0, F_{LL_i} < 0, F_{KK_i} < 0, \) and \( F_{LL_i}F_{KK_i} - (F_{KL_i})^2 \geq 0 \) for all \( (L_i, K_i) \in \mathbb{R}_{++}^2 \).
The supply of capital is fixed at some level $\bar{K}$. Full employment of capital therefore requires that

$$K_1 + K_2 = \bar{K} \quad (1)$$

always holds. We denote the rental price of capital by $r$ and the gross wage by $w$. Private employers pay a fraction $t$ of wages to the government as a wage tax. Workers, thus, earn a net wage of $w(1-t)$ per unit of labor supply.

Sector 2 is always privately run and operated as to maximize profits

$$\Pi_2 = F^2(L_2, K_2) - r \cdot K_2 - w \cdot L_2.$$ Profit maximization requires that marginal productivities equal factor prices:

$$F^2_L(L_2, K_2) = w \quad (2)$$

$$F^2_K(L_2, K_2) = r. \quad (3)$$

Sector 1 can be either government-operated or privately-run. We assume that the sector has to provide a certain exogenously given level $\bar{F}^1$ of output:$^5$

$$F^1(L_1, K_1) \geq \bar{F}^1. \quad (4)$$

We assume that production in sector 1 is organized in a cost-minimizing manner. This is a prerequisite for profit maximization and therefore appears to be an appropriate hypothesis if the sector is in private hands. Assuming cost efficiency in the public sector might be more controversial, given ample evidence for governmental slack. By requiring cost efficiency we deliberately rule out all reasons for contracting out that might arise from an inefficient organization of the public sector.

- If the sector is privately-run, then the cost-minimization problem reads as:

$$\min_{L_1, K_1} \left\{ rK_1 + wL_1 | F^1(L_1, K_1) \geq \bar{F}^1 \right\}. \quad (5)$$

To assess labor costs, the private firm uses the gross, tax-inclusive wage rate. The first-order conditions for cost efficiency are given by:

$$\frac{F^1_L(L_1, K_1)}{F^1_K(L_1, K_1)} = \frac{w}{r}$$

and the output requirement (4).

$^5$We perceive of $\bar{F}^1$ as being politically determined; we do not attempt to derive an optimal output level of sector 1.
• If the sector is government-operated and its recruiters recognize that taxes paid by government entities cancel out upon consolidation of all government accounts, cost minimization can be based on the net wage rate $w(1-t)$:

$$\min_{L_1,K_1} \{ rK_1 + w(1-t)L_1 | F^1(L_1,K_1) \geq \bar{F}^1 \}.$$ 

The first-order conditions for cost efficiency in such an effectively tax-exempt government sector are given by:

$$\frac{F^1_1(L_1,K_1)}{F^1_K(L_1,K_1)} = \frac{w(1-t)}{r}$$

and, again, the output requirement (4).

Generalizing (5) and (6), we introduce a parameter $\alpha \in [0,1]$ to measure the extent to which the government has, or its authorities that recruit staff into government services perceive the government to have, a tax-induced cost advantage over the private sector: $\alpha = 1$ means that public sector recruiters fully see through the accounting mechanisms of the consolidated government budget while $\alpha = 0$ means that wage taxes are fully part of labor costs to the hiring government agency; this would be equivalent to contracting out the production of good 1 to the private sector. Variable $\alpha$ may also reflect the degree to which employees in government-run entities are exempt from taxes or contributions that are collected in the private sector.

With some leap of faith in the existence of aggregate production functions, one might also interpret $\alpha$ as the fraction of sector 1 that is government-operated. Such an interpretation might, e.g., be appropriate for the case of public transport when only parts of the network are operated through private companies. However, this interpretation requires that production in sector 1 can be additively aggregated from a number of microproduction functions – which will only be possible under quite restrictive conditions (cf., e.g., Felipe and Fisher, 2003).

The variable $\alpha$ also allows for an interpretation in terms of a federalist structure. Suppose, e.g., that sector 1 is run by local municipalities. Then $(1-\alpha)$ might be viewed as that part of wage taxes that directly flows to municipalities and that would therefore not be regarded as part of the labor costs by local decision makers, while $\alpha$ denotes tax revenues that first flow to a higher tier in the federal system in order to be returned, in a lump-sum fashion, to the local level afterwards.
Using $\alpha$, the cost minimization procedure can be written as:

$$\min_{L_1,K_1} \left\{ rK_1 + w(1 - \alpha t)L_1 | F^1(L_1, K_1) \geq \bar{F}^1 \right\}$$

(7)

and the attending first-order condition (apart from the output constraint) reads:

$$\frac{F^1_L(L_1, K_1)}{F^1_K(L_1, K_1)} = \frac{w(1 - \alpha t)}{r}.$$  

(8)

Denote the solutions to (7) by $K_1(\alpha)$ and $L_1(\alpha)$. Similarly, we might index all other variables by $\alpha$. From a mathematical perspective, the advantage from using continuous $\alpha$ rather than a dichotomous $\alpha \in \{0, 1\}$ lies in making the whole problem differentiable.

Equation (8) together with the output constraint immediately implies that labor input in sector 1 is higher and consequently capital input is lower the larger is $\alpha$, meaning that the labor intensity is ceteris paribus higher when sector 1 is government-owned rather than when it is contracted out. Below we will show that this pattern also emerges in a general equilibrium.

### 3.2 Households and Labor Market

The economy is populated by a single individual with preferences over the consumption of goods 1 and 2 and over leisure. We assume that the solution to the individual’s utility maximization problem gives rise to a supply function for labor that increases in the net wage rate:

$$L_S = L_S[w(1 - t)]$$

with $L'_S[w(1 - t)] > 0$. Denote by

$$\eta^S := L'_S \cdot \frac{w(1 - t)}{L_S}$$

the elasticity of labor supply with respect to the net wage. In a labor market equilibrium the labor intake of the two sectors equals labor supply:

$$L_1 + L_2 = L_S[w(1 - t)].$$

### 3.3 Government

Our model is closed by the government budget constraint. Fiscal needs arise from the fact that good 1 is provided to the citizens free of (direct) charge:
• We assume that if sector 1 is privately-run, the government purchases the output from there. The price for output $F_1$ has at least to cover the costs of production $r \cdot K_1(0) + w \cdot L_1(0)$. Government revenues stem from taxes on employment in the two sectors and amount to $t \cdot w \cdot (L_1(0) + L_2(0))$. A balanced budget therefore requires

$$rK_1(0) + w(1 - t)L_1(0) = twL_2(0).$$

• If production of good 1 takes place in a government that fully sees through its budgetary mechanics, the costs of production amount to $rK_1(1) + w(1 - t)L_1(1)$ from (6). Tax revenues only come from labor employed in sector 2, such that the budget constraint reads:

$$rK_1(1) + w(1 - t)L_1(1) = twL_2(1)$$

which is the same as in the previous case (noting, of course, that the input variables may take on different values).

Generalizing with the use of $\alpha$, this does not change; the government budget always has the form:

$$rK_1(\alpha) + w(1 - t)L_1(\alpha) = twL_2(\alpha)$$

or, upon using that $r = F_2^2$ and $L_2 = L_S - L_1$,

$$F_2^2 \cdot K_1(\alpha) + w \cdot (L_1(\alpha) - tL_S) = 0. \quad (9)$$

### 3.4 Reduced Form

Summarizing (1) to (4), and incorporating (8) and (9), the equilibrium of the economy can be characterized by the following system of equations:

$$F_1^1(L_1, K_1) \cdot F_2^2 (L_S[w(1 - t)] - L_1, \bar{K} - K_1)$$

$$- F_1^1(L_1, K_1) \cdot w \cdot (1 - \alpha t) = 0 \quad (10)$$

$$F_1^1(L_1, K_1) - F_1^1 = 0 \quad (11)$$

$$F_2^2 (L_S[w(1 - t)] - L_1, \bar{K} - K_1) - w = 0 \quad (12)$$

$$F_2^2 (L_S[w(1 - t)] - L_1, \bar{K} - K_1) \cdot K_1 + w \cdot (L_1 - tL_S[w(1 - t)]) = 0. \quad (13)$$
Equation (10) is the cost-efficiency condition for the production of good 1, equation (11) is the minimal-output requirement for that good, equation (12) is the condition for profit-maximizing labor input in the production of good 2, and equation (13) is the government budget constraint. Equations (10) through (13) have to be solved for the variables $L_1, K_1, w$, and $t$ from which all other endogenous variables of the model can then be determined. The solution can be parametrized by $\alpha$.

Observe that an efficient allocation of factors of production requires that the marginal rates of factor substitution are equalized across sectors:

$$\frac{F^1_1}{F^1_K} = \frac{F^2_1}{F^2_K}.$$  

This only holds in a private economy, i.e., if $\alpha = 0$. As the output of good 1 is exogenously fixed, (14) means that the available inputs are used such as to maximize the output of good 2.

4 Comparative Statics

We will henceforth assume that labor and capital are gross complements in both sectors: $F^i_{KL}(L_i, K_i) \geq 0$ for $i = 1, 2$. This assumption, which is in accordance with empirical evidence on labor demand at least for skilled labor (see Borjas, 2004, ch. 4), facilitates the technical analysis without being strictly necessary. We derive comparative statics of (10) to (13) with respect to the tax advantage $\alpha$ of the government sector and first consider the case of a variable labor supply. In the Appendix, we prove

**Proposition 1** Suppose that labor supply is strictly increasing in the net wage, $L'_S[w(1-t)] > 0$, and that the equilibrium of the economy exhibits Hicksian stability. Assume further that

- the elasticity of labor supply does not exceed $(1-t)/t$, or
- the tax rate $t$ is small.

Then labor input in sector 1 increases and capital input decreases upon an increase in $\alpha$. The effects on the equilibrium gross wage and the tax rate are generally ambiguous.

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$^6$One could also nationalize sector 2 to obtain production efficiency. However, this would define away the problem we are interested in.
The condition \( \eta^S \leq (1 - t)/t \) in Proposition 1 is equivalent to the requirement that the tax elasticity of labor supply is, in absolute terms, less than unity:

\[
\eta^S_t := \frac{\partial L_S[w(1 - t)]}{\partial t} \cdot \frac{t}{L_S} = -\frac{t}{1 - t} \cdot \eta^S \geq -1.
\]

This is in harmony with stylized facts on labor supply elasticities. Moreover, if this condition were not satisfied, an increase in \( t \) would ceteris paribus reduce wage tax revenue \( twL_S[w(1 - t)] \).

Next consider the case of a fixed labor supply. Setting \( L'_S = 0 \) in equations (24) to (27) in the proof of Proposition 1 immediately leads to

**Proposition 2** Suppose that labor supply is constant, \( L'_S = 0 \), and that the equilibrium exhibits Hicksian stability. Then labor input in sector 1 increases and capital input decreases upon an increase in \( \alpha \). The higher \( \alpha \), the higher the gross wage, while the effect of \( \alpha \) on the equilibrium tax rate is generally unclear.

Propositions 1 and 2 imply that labor input in sector 1 is higher and capital input is lower if the sector is government-run rather than if it is privately-run. Hence, the partial effect that public production is more labor-intensive than production of the same output in the private sector, carries over to a general equilibrium.

5 Welfare Analysis

5.1 The Potential Trade-off

There is a potential trade-off when it comes to choose how production in sector 1 should be organized. Only if the sector is privately-run (\( \alpha = 0 \)), production efficiency in the sense of (14) will be achieved. On the other hand, if the net wage \( w(1 - t) \) and, thus, labor supply are higher when a government agency with some labor tax advantage (\( \alpha > 0 \)) operates production in sector 1 it may be possible to augment output in sector 2 in spite of an intersectorially inefficient factor allocation.

An instructive way to view this trade-off is in terms of an Edgeworth box for the production possibilities of the economy:

Figure 1 goes here.
Figure 1 depicts production possibilities for $\alpha = 0$ (fully private economy). The economy will be in a point like $A$: Production is efficiently organized — the isoquants of the production functions in sectors 1 and 2 are tangent. The output level in sector 2 is $F^2(0)$. The second isoquant for good 2 in Figure 1 represents a higher but unattainable output level.

Figure 2 goes here.

Figure 2 depicts production possibilities in the case of $\alpha > 0$ (mixed economy), provided that this leads to an increase in the net wage. As a consequence, the width of this Edgeworth box is larger than in Figure 1, reflecting the increase in labor supply. The previously unattainable output level $F^2(\alpha)$ becomes feasible now. However, the economy ends up in a point like $B$: Sectors 1 and 2 face different factor-price ratios, and consequently isoquants at the equilibrium output levels will intersect rather than being tangent to each other. In a nutshell, the potential difference between a fully private economy (Figure 1) and a mixed economy (Figure 2) boils down to operating efficiently in a “small” Edgeworth box and operating inefficiently in a larger one.

5.2 Formal Analysis

From Propositions 1 and 2, it is unclear whether a case as in Figure 2 can at all emerge. As it requires an increase in labor supply, we can definitely rule it out whenever labor supply is constant:

**Proposition 3** If labor supply is fixed ($L'_S \equiv 0$), then a private economy (i.e., $\alpha = 0$) is optimal.

With variable labor supply (and an increasing net wage), we need to weigh the two elements in the trade-off between intersectoral efficiency and a larger factor supply. Welfare analysis, thus, gets a bit more involved.

Underlying our model is a representative household with preferences over the consumption of goods 1 and 2 and leisure. We represent these preferences by a standard quasi-concave utility function

$$U = U(c_1, c_2, -L_S)$$
where all partial derivatives are positive. Consumption of good 1 equals the exogenous output in sector 1. The household maximizes utility subject to a budget constraint

\[ c_2 \leq y + w(1 - t)L_S \]  

(15)

where \( y \) denotes income from sources other than labor supply (i.e., capital income and profits, if any, from sector 2 and capital income from sector 1). Optimal labor supply satisfies the first-order condition:

\[ w(1 - t)U_2 - U_3 = 0. \]  

(16)

Given that output of good 1 is fixed, a change in \( \alpha \) will then lead to an increase in utility if and only if

\[ \frac{dU}{d\alpha} = U_2 \cdot \frac{dc_2}{d\alpha} - U_3 \cdot \frac{dL_S}{d\alpha} = U_2 \cdot \left( \frac{dc_2}{d\alpha} - w(1 - t) \cdot \frac{dL_S}{d\alpha} \right) > 0 \]  

(17)

where we used (16).\(^7\) In an equilibrium, consumption of good 2 equals production of that good, i.e., \( c_2 = F^2(K_2, L_2) \). If we vary \( \alpha \), output in sector 2 is affected as follows:

\[
\frac{dF^2}{d\alpha} = F^2_L \cdot \frac{dL_2}{d\alpha} - F^2_K \frac{dK_1}{d\alpha} \\
= \left( \frac{F^2_K}{F^2_L} \right) \frac{dL_1}{d\alpha} + F^2_L \frac{dL_S}{d\alpha} = w \left( \frac{dL_S}{d\alpha} - \alpha t \cdot \frac{dL_1}{d\alpha} \right) \\
= w \left( \frac{dL_S}{d\alpha} - \alpha t \cdot \frac{dL_1}{d\alpha} \right). 
\]  

(18)

Here we invoked \( dK_1/d\alpha = -(1-\alpha t)(F^2_L/F^2_K)(dL_1/d\alpha) \). If labor supply is fixed (\( dL_S = 0 \)), then output in sector 2 decreases whenever the government runs production of good 1. A negative impact of \( \alpha \) on labor supply would acerbate the reduction of output in sector 2; only with a positive impact on labor supply can the effect be turned around.

Plugging (18) into (17), we find that welfare improves with an increase in \( \alpha \) if and only if:

\[ \frac{dL_S}{d\alpha} > \alpha \cdot \frac{dL_1}{d\alpha}. \]  

(19)

\(^7\)Observe that this line of reasoning does neither involve nor require any analysis of how the separate components of the household’s income (wage and interest income and profits) are affected. Our arguments are purely based on a utility assessment of the changes in the consumption of good 2 and labor supply in the general equilibrium of the economy. Production feasibility, changes in the interest rate and capital income are, thus, always implicitly but properly accounted for.
Condition (19) links the increase in the labor supply to the tax wedge and the distortion in the factor mix which arises from the tax advantage of the public sector. The left-hand side of (19) is the increase in the total labor supply as a result of the public sector tax advantage, corresponding to an increase in the size of the Edgeworth box. The right-hand side of (19) relates to the distortion inside the Edgeworth box. The distortion in the allocative efficiency caused by a change in the demand for labor in sector 1 is increasing in the tax advantage of the public sector, measured by \( \alpha \). Therefore, a higher tax wedge requires a proportionally larger increase in the aggregate labor supply in order to improve welfare.

An immediate consequence of this observation is that, starting from a fully private economy (\( \alpha = 0 \)), an increase in \( \alpha \) will be welfare-improving if and only if it leads to an increase in labor supply or, which is the same, to an increase in the net wage.

Observe that (19) can equivalently be written as

\[
\frac{d[w(1-t)]}{d\alpha} > \frac{\alpha}{L_s} \frac{dL_1}{d\alpha} \tag{20}
\]

Given that \( dL_1/d\alpha > 0 \) is plausible from Proposition 1, condition (20) conveys that a welfare improvement is possible only if the net wage increases sufficiently sharply upon an increase in \( \alpha \) (or, conversely, if outsourcing production of good 1 from the government to the private sector leads to a sufficiently large drop in net wages). Whenever outsourcing of sector 1 would decrease the wage rate it can never be optimal to do so: the left-hand side in (20) is always larger than zero. We sum this up in

**Proposition 4** Fully private production in sector 1, i.e., reducing \( \alpha \) to zero, can never be optimal if it leads to a decrease in net wages. Increasing \( \alpha \) is welfare-improving if it leads to a sufficiently large increase in the after-tax wage.

Proposition 4 is a second-best result: With variable labor supply, wage taxation is distortionary in the sense that the marginal rate of substitution between leisure and the consumption of good 2, \( U_3/U_2 = w(1-t) \), does not equal the marginal productivity of labor in the production of good 2, \( F_2^L = w \). It may then not be optimal to aim at production efficiency. Violations of condition (14) can be induced by giving sector 1 a tax advantage over sector 2, which in our framework is tantamount to having this sector
(partly) government-operated. One visible impact of such a policy is then a higher labor intensity of the public sector, relative to what a private enterprise would choose.

Proposition 4 states conditions such that full contracting out \((\alpha = 0)\) is not optimal. This does, however, not imply that a monolithic social planner with \(\alpha = 1\) should take over production of sector 1. Rather, intermediate values of \(\alpha\) might dominate the polar cases. As outlined above, one way to think of such intermediate values is in terms of partial contracting out or of a mixed personnel structure (both civil servants and normal employees). Under the latter interpretation, Proposition 4 conveys that entirely staffing sector 1 with normal employees (represented by \(\alpha = 0\)) is not optimal, but that to have some tax-favored civil servants \((\alpha > 0)\) might actually be preferable. Alternatively, our results can be interpreted as an efficiency argument in favor of a federal structure in which lower-level governments receive a certain fraction of (centrally administered) wage tax revenues. This gives them a potentially welfare-improving tax advantage over the private sector. As a 100%-tax advantage \((\alpha = 1)\) will, in general, not be optimal, our results also suggest an efficiency explanation for a certain degree of fiscal churning in which the federal government would collect a share \((1 - \alpha)\) of the tax revenue and return it to lower-level governments as lump-sum transfers.

### 5.3 The Role of the Labor Supply Elasticity

From Proposition 4, deviations from production efficiency can only be worthwhile if they go along with a sufficiently strong increase in the net wage. To assess the wage effect, we combine (26) and (27), where \(\beta > 0\) is defined in (23) in the Appendix:

\[
\frac{d[w(1 - t)]}{d\alpha} = (1 - t) \cdot \frac{dw}{d\alpha} - w \cdot \frac{dt}{d\alpha} = -\beta \cdot w \cdot \left(-2(1 - t)L^S_F L^2_F K^1_F + (L^S - L_1) \cdot \left[F^1_K F^2_L - F^1_K F^2_K L\right] + F^1_K \cdot \left[w - F^2_K K^1_L\right] - F^1_L \cdot \left[F^2_K K^1_F F^2_K L\right]\right) = -\beta \cdot w \cdot \left(-2(1 - \alpha t) \cdot \eta^S \cdot L^S_F L^2_F K^1_F + (L^S - L_1) \cdot \left[F^1_K F^2_L - F^1_L F^2_K L\right] + w \cdot \left(\frac{\alpha t F^1_K}{F^1_L F^2_K} + K^1_L \cdot \left[F^1_K F^2_K - F^1_K F^2_K L\right]\right)\right) \tag{21}
\]

To arrive at the final line of (21), we made use of \(F^1_K w - F^1_L F^2_K = F^1_K \cdot \left[w - \tau F^1_L / F^1_K\right] = F^1_K \cdot \omega t\) which stems from (2), (3), and (7).
Combining (20) and (21), one sees opposing forces at work: For a welfare improvement, (21) must, according to (20), exceed \((dL_1/d\alpha) \cdot (\alpha/L_S')\) which is positive whenever \(dL_1/d\alpha > 0\). Expression (21) is smaller and, thus, more likely to be negative if labor supply elasticity is higher (the cofactor of \(\eta^S\) is negative). Thus, for high values of \(\eta^S\), (20) cannot be satisfied. On the other hand, if \(\eta^S\) is getting very small, the right-hand side of (20) exceeds all bounds, making it again impossible for the condition to hold.\(^8\)

This observation renders general results unobtainable. However, from (20), an increase in the net wage suffices to make deviations from \(\alpha = 0\) worthwhile. From (21), the wage rate will increase for positive, but small labor-supply elasticities. We summarize:

**Proposition 5** *Fully private production in sector 1 (\(\alpha = 0\)) can never be optimal if the labor supply elasticity is positive but small.*

Recall that none of our observations hinges on the assumption that only labor is taxed. Without qualitatively affecting any of our results we could allow for taxes on capital as long as they are lower than taxes on labor and alone do not suffice to cover the financial needs of the government. The crucial feature is that the tax structure we impose is not an optimal one for the economy. This generates a bias in favor of having two differently taxed sectors (i.e., a mixed economy) and a distorted production structure. In line with the earlier literature on optimal taxation and public production, full optimality in our set-up could be reached by leaving both sectors in private hands and taxing only (inelastically supplied) capital in a private economy. We consider the assumption that the economy operates under a sub-optimal tax structure not being too far-fetched; in that case, some intersectoral distortions may be worthwhile.

6 Conclusion

In this paper, we analyze the relationship between public production and taxation from a general-equilibrium perspective. Several publicly provided goods and services, like hospitals, schools, and public transportation, can be produced privately even if they are ultimately financed by the government. Empirical evidence suggests that contracting out

\(^8\)This point could already be seen from (19): if the elasticity of labor supply is too small then (19) cannot hold as its left-hand side approaches zero, while its right-hand side does not.
such activities tends to result in a leaner workforce and increases the capital intensity in their production. We argue that this need not be viewed as evidence of slack in public production. In a consolidated government budget, the government “pays taxes to itself”. As a consequence, the government sector has a cost advantage over the private sector for the factor that is taxed relatively more heavily. As – in our model, but also in most countries – labor is taxed more heavily than capital, the government would then optimally organize production in a more labor-intensive way than a private firm.

Moreover, different factor price ratios and, therefore, different marginal rates of technical substitution in public and private production are not necessarily an evil. We identify a key trade-off in deciding whether to contract out government activities or not. On the one hand, different factor prices faced by public and private entities distort allocative efficiency. On the other hand, a higher labor-intensity of government-run activities may serve as a countervailing distortion in the presence of distorting wage taxation. Contracting out government production and then letting the government re-purchase the output may, under certain circumstances, result in a decrease in equilibrium net wages and, thus a reduction in labor supply. If the associated reduction in production possibilities is sufficiently severe it may well prove beneficial to incur the production inefficiencies in a mixed economy, compared to a production-efficient economy with smaller production possibilities. Given that these effects are driven by changes in labor supply, we argue that fully private production is never optimal with positive but low labor supply elasticities, as in Europe.

For public production that takes place at lower-level jurisdictions in federations our analysis also suggests an efficiency argument for the otherwise puzzling phenomenon of fiscal churning where the central government collects a share of tax revenues and returns it as lump-sum transfers to all lower-level jurisdictions, and not just to poorer ones. Fiscal churning can be understood as an attempt to influence the price ratios that lower-level jurisdictions effectively face when financing their production activities.

There are several ways in which our analysis could be extended. One might consider a small open economy where the rental rate of capital is exogenously given. Moreover, one could dispense with the assumption that governments are price takers in the factor markets. While this is an appropriate assumption in the case of local municipalities and individual government agencies, it is implausible for the central level of government as a
whole. With reasonable assumptions about labor supply elasticities and production functions in different sectors, it would be possible to estimate the optimal level of contracting out. Finally, the assumption of an exogenously given output in the government-financed sector could be loosened. These extensions, as well as empirical testing of the predictions and evaluation of quantitative importance of our findings, are left for further research.

Appendix: Proof of Proposition 1

**Proof:** Differentiating (10) to (13) with respect to $\alpha$ yields the following system of equations:

\[
\begin{pmatrix}
  a_1 & a_2 & a_3 & a_4 \\
  b_1 & b_2 & 0 & 0 \\
  c_1 & c_2 & c_3 & c_4 \\
  d_1 & d_2 & d_3 & d_4
\end{pmatrix}
\begin{pmatrix}
  dL_1 \\
  dK_1 \\
  dw \\
  dt
\end{pmatrix}
= \begin{pmatrix}
  -wtF_K^1 \\
  0 \\
  0 \\
  0
\end{pmatrix}
\cdot d\alpha
\]

with

\[
\begin{align*}
  a_1 &= F_{LL}^1 F_K^2 - F_{LK}^1 F_{KK}^2 - F_{KL}^1 w(1 - \alpha t) < 0 \\
  a_2 &= F_{KL}^1 F_K^2 - F_{LK}^1 F_{KK}^2 - F_{KK}^1 w(1 - \alpha t) \\
  a_3 &= (1 - t)F_{LL}^1 F_{KK}^2 L_S' - F_{KK}^1 (1 - \alpha t) \\
  a_4 &= -wF_{KL}^1 F_{KK}^2 L_S' + F_{KK}^1 w\alpha \\
  b_1 &= -F_{LL}^1 \\
  b_2 &= -F_{KK}^1 < 0 \\
  c_1 &= -F_{LL}^2 \\
  c_2 &= -F_{LK}^2 \\
  c_3 &= F_{LL}^2 L_S'(1 - t) - 1 < 0 \\
  c_4 &= -F_{LL}^2 L_S' w \\
  d_1 &= w - K_1 F_{KL}^2 \\
  d_2 &= F_K^2 - K_1 F_{KK}^2 \\
  d_3 &= -tL_S + L_1 - wtL_S'(1 - t) + K_1 F_{KL}^2 L_S'(1 - t) \\
  d_4 &= -wL_S + w^2 tL_S' - K_1 F_{KL}^2 L_S' w.
\end{align*}
\]
Denote the matrix on the left-hand side of (22) by $A$. Observe that we arranged the matrix such that the diagonal elements $a_1$, $b_2$, and $c_3$ are negative. Also $d_4$ will be negative for small values of $t$ or, as long as $t \leq 0.5$ if the elasticity of labor supply is below unity.

In order for the system to be perfectly stable (i.e., stable in the Hicksian sense), $A$ must then be negative semi-definite (see Takayama, 1985, pp. 313ff). In particular, $\det A > 0$ — which we will henceforth assume. For sake of abbreviation define:

$$\beta := \frac{w F^1_K}{\det A} > 0,$$

where the positive sign prevails when $A$ is stable. Now apply Cramer’s Rule to (22):

$$\frac{dL_1}{d\alpha} = \beta \cdot (c_3d_4 - c_4d_3) \cdot F^1_K = \beta \cdot w \cdot (L_S - L'_Swt + L'_S \cdot [-F^2_{LL}(L_S - L_1) + K_1F^2_{KL}]) \cdot F^1_K$$

$$= \beta \cdot w \cdot \left( L'_S \cdot \left[ 1 - \eta^S \cdot \frac{t}{1 - t} \right] + L'_S \cdot \left[ -F^2_{LL}(L_S - L_1) + K_1F^2_{KL} \right] \right) \cdot F^1_K \tag{24}$$

$$\frac{dK_1}{d\alpha} = -\beta \cdot (c_3d_4 - c_4d_3) \cdot F^1_L = -\frac{F^1_L}{F^1_K} \cdot \frac{dL_1}{d\alpha} \tag{25}$$

$$\frac{dw}{d\alpha} = -\beta \cdot (b_2(c_4d_1 - c_1d_4) + b_1(c_2d_4 - c_3d_4)) \tag{26}$$

$$= -\beta \cdot w \cdot \left( L'_S \cdot \left[ \Gamma - F^1_L F^2_K F^2_{LL} \right] + L'_S \cdot \left[ F^1_K F^2_{LL} - F^1_L F^2_{KL} \right] \right)$$

$$\frac{dt}{d\alpha} = -\beta \cdot (b_2(c_1d_3 - c_3d_1) + b_1(c_3d_2 - c_2d_3)) \tag{27}$$

where we defined:

$$\Gamma := w(1-t)F^1_K F^2_{LL} + F^1_L \cdot \left( K_1 \cdot \left[ F^2_{KK} F^2_{LL} - (F^2_{KL})^2 \right] + wtF^2_{KL} \right),$$

which is of ambiguous sign. Given the assumptions mentioned in the proposition, the signs of (24) and (25) turn out as asserted, while the signs of (26) and (27) remain unclear in general.

**References**


Figure 1
Figure 2