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Abstract

Evolutionary stability is a necessary condition for imitative dynamics of policy learning and innovation to come to a rest. We apply this concept to profit tax competition in a regime where a common and consolidated profit tax base for multi-jurisdictional firms is divided among governments by means of formulary apportionment. In evolutionary play, governments exhibit aggregate-taking behavior: when comparing their performance with others, they ignore their impact on the consolidated tax base. Consequently, evolutionarily stable tax rates are less efficient than tax rates in best-response tax competition.

Keywords: Imitation and learning, Tax competition, Formulary apportionment, CCCTB.

JEL Classification: H77, H25, F23, C73.

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1 Introduction

In 2016, the European Union (EU) launched another effort to abandon its fragmented collection of 28 competing tax laws and separate accounting for corporate income taxation. Building on earlier suggestions, the recent proposal is to use a method of formulary apportionment (FA) applied to a common consolidated corporate tax base (CCCTB) that aggregates the EU-wide profits of multinational groups which operate firms in EU member states.[1] While a number of federal countries – most prominent: Canada, the US, Switzerland and Germany – have since long been using apportionment methods to share between their jurisdictions the base for profit taxes (e.g., Siu et al., 2014; Hellerstein and McLure, 2004), moving to the formulary apportionment of a consolidated aggregate tax base for corporate income would be a big step into an unfamiliar tax territory for the EU.

If the proposal came true, how should governments set their tax rates – which, for reasons of national sovereignty in the EU, remain policy instruments in their hands? Almost all (theoretical) research on formulary apportionment presupposes that tax rates would be set optimally, i.e., as payoff-maximizing, best responses to what other governments do. Corporate income taxation in an economically integrated area is, thus, viewed as a non-cooperative game with the Nash equilibrium as the predicted outcome.

The informational requirements underlying best-response play are unrealistically demanding. In particular, for optimization governments need to fully know the mapping

[1] Explorations into the CCCTB started with European Commission (2001), came to a halt in 2008, and rather unsuccessfully reappeared on the policy agenda of European Commission and European Parliament in 2011. In October 2016, the European Commission proposed another set of Directives pushing forward unitary tax rules for business operations across the EU European Commission (2016). If adopted, a common corporate tax base (CCTB) becomes mandatory, as of January 2019, for EU companies belonging to a group with a consolidated turnover exceeding EUR 750 million. In a second step, the cross-border consolidation of profits and losses will become mandatory from January 2021 onwards, transforming the CCTB into a common consolidated corporate tax base (CCCTB) with formula apportionment of taxable group profits based on three equally weighted factors comprising labor (half allocated to payroll and half to number of employees), assets (all fixed tangible assets, but not intangible or financial assets), and sales (by destination). The rationale of the CCCTB is to remedy the multiple defects of income taxation for multinationals in the EU that mainly originate from the co-existence of so many different and inconsistent tax systems. These defects result in distorted factor allocations, high administrative and compliance costs, tax avoidance through cross-border profit shifting and transfer pricing, thin capitalization etc. which consolidation and formula apportionment are hoped to remedy (European Commission, 2001, 2003, 2006, 2011, 2016; Devereux, 2004; Sørensen, 2004; Weiner, 2006; Fuest, 2008; Avi-Yonah and Benshalom, 2011).

from tax policies to (expected) payoffs. This requires that the "true" economic structure is well-understood and parametrically accurate, including mobility elasticities, policy impacts, international spillovers, strategic interaction effects etc. The novelty of formulary apportionment in the EU, but possibly also the complexity of formulary apportionment in general render assumptions that governments possess enough knowledge to craft best-response policies implausible (Arel-Bundock and Parinandi, 2017).[2]

Alternative approaches to fiscal decentralization precisely take such a lack of knowledge as their starting point. For them, the rationale for running economic systems in a decentralized way rather than through top-down social planning is that knowledge on how the economy is working is not available or, at least, insufficient to run optimization routines in government finance. Centralized, one-size-fits-all policies would then carry a high risk of damaging the entire system by bad policy choices. By contrast, fiscal decentralization limits eventual damages to a smaller scale. More importantly, it works like a set of laboratories; fiscal interaction is a discovery procedure that evolves through experimentation, learning-by-doing and the imitation of good ideas (Hayek, 1978; Oates, 1999, 2008; North, 1981; Kollman et al., 2000; Vanberg and Kerber, 1994; Vihanto, 1992; Baybeck et al., 2011).

In this paper, we analyse tax competition with formulary apportionment of a common tax base from such an evolutionary perspective. Specifically, we have in mind "best-practice"-processes like this: governments are in repeated interaction over a long period of time. They observe each others' policies (tax rates, say) and the attending outcomes (tax revenues). They do not comprehend the "true" economy; when considering changes in their policy, governments most of the time adopt the policy (i.e., set the rate of the corporate income tax) that, in the previous period, worked best in the set of all countries

[2]Empirical knowledge on firm behaviour, tax planning, and the effects of changes in tax rates and sharing mechanisms under formulary apportionment is scarce, however, relative to separate accounting. In addition, studies are often confined to (sub-)national contexts in the USA (e.g., Edmiston and Arze del Granado, 2006), Canada (e.g., Mintz and Smart, 2004), or Germany (e.g., Riedel, 2010), making transfers to international settings problematic. Lacking historical data, predictions on tax revenues under FA are based on simulation models (Devereux and Loretz, 2008; Fuest et al., 2007; Bettendorf et al., 2010; Spengel et al., 2012).

(imitate-the-best). Occasionally, governments try new policies (experimentation). Such experiments will be copied by other governments and thus spread out if they are "successful"; otherwise they will be discarded and undone. "Successful" is understood in terms of *relative comparisons*: only if deviators (i.e., experimenting, innovating jurisdictions) earn higher payoffs than non-deviators will their policies spread out. In the long run, so the theory of laboratory federalism hopes, this process of innovation, learning, and mimicking will lead, in a decentralized way, to the adoption of superior, if not efficient policies everywhere.

Building on game-theoretical results on learning and imitation dynamics (Fudenberg and Imhof, 2006; Alós-Ferrer and Schlag, 2009), recent attempts to formalize laboratory federalism argue that the iterated process of policy experimentation and imitation will eventually stochastically converge to so-called *evolutionarily stable strategies (ESS)* of the underlying static stage game. A policy is called an ESS if, once it has been adopted by all governments, it cannot be successfully taken over by the emergence of an alternative policy (Schaffer, 1988) in the sense that a deviator would be better off than a non-deviator.[3] In finite-player games – like fiscal federalism or tax competition –, an ESS is generally neither a Nash equilibrium strategy nor efficient. In fact, in large classes of games ESS leads to "competitive" outcomes (Alós-Ferrer and Ania, 2005): players behave as if they had no impact on the aggregates of the economy, i.e., governments ignore all effects of their policies on economic variables that are common to the entire set of governments. The rationale is that, if governments base policy choices on payoff comparisons among themselves, all aggregate effects are irrelevant – precisely because they affect every player in the same way. Such "aggregate-taking behavior" leads, in many cases, to extremely sharp races to the bottom or over the top, amplifying the inefficiencies already prevailing in best-response play.[4]

[3]An additional piece of motivation comes from the fact that an ESS is the predicted Nash equilibrium strategy when players aim at maximizing their relative (rather than absolute) payoffs. For models of fiscal competition, this captures yardstick competition and other policies inspired by keeping up with other governments.

[4]Observations of highly inefficient ESS have been made for several standard models of fiscal decentralization such as capital tax competition (Sano, 2012; Wagener, 2013; Philipowski, 2015), infrastructure

In tax competition with formulary apportionment, such inefficiencies arise from spillovers generated both by the apportionment method and the definition of the tax base. Their interplay also shapes strategic incentives in best-response tax competition, which may be of the "race-to-the-bottom" type as well as of the "race-over-the-top" type (e.g., Sørensen, 2004; Eggert and Schjelderup, 2003; Wellisch, 2004; Pethig and Wagener, 2007; Pinto, 2007; Eichner and Runkel, 2011, 2012).[5]

In this paper we, first, find that the ESS of tax competition with tax-base apportionment only depends on the apportionment formula – and not (so much)[6] on the definition of the consolidated tax base. Second, comparing the ESS, the Nash equilibrium and the cooperative outcome of the tax competition game, all three solutions generally differ. If the fiscal game exhibits positive [negative] externalities, then the ESS tax rate is lower [higher] than the Nash equilibrium rate, which in turn is lower [higher] than the efficient tax rate. Evolutionary play, thus, exacerbates the inefficiencies of best-response play. In the knife-edge case where the tax base and apportionment are designed in such a way that fiscal externalities are internalized, then the ESS and the Nash equilibrium are the same – and both are efficient. In such an ideal situation, the outcome of fiscal interaction would be independent of how governments play the fiscal game: as sophisticated rational agents or as behavioral and learning agents.

Our paper adds to the literature on the CCCTB-project of the EU, where evolutionary play and relative payoff concerns have not yet been studied. Our approach and findings are, however, not tied to this project and its uncertain prospects; they carry over to other tax settings with formulary apportionment as well. They in fact generally apply to all situations where an aggregate tax base or, more broadly, a common pool resource is to be divided among several governments and the value of the resource and/or the division

competition (Wagener, 2013) or decentralized redistribution (Ania and Wagener, 2016).

[5] Tax competition with formulary apportionment indeed looks quite different than "standard" tax competition with separate accounts, the status-quo of EU corporate income taxation. Separate accounting encourages firms to shift taxable profits into low-tax countries, implying positive fiscal cross-border externalities of corporate income taxes (Nielsen et al., 2010).

[6] The definition of taxable profits only matters to the extent that it affects the elasticity of total investment in the economic area. With a fixed capital stock it would not matter at all.

method are endogenous to the players' choices (Konrad, 2008). To demonstrate this, we start our exposition in Section 2 with a general model of a shared common tax resource, which only in Section 4 will be fleshed out to a model of tax competition with a CCCTB.

Our paper adds to a growing literature on policy diffusion and learning by governments. Different than the tax competition literature, which views tax policy interdependence as solely strategic, policy diffusion scholars suggest a variety of interdependence mechanisms, including learning, imitation, competition, coercion, common norms, and taken-for-grantedness, in addition to competition (Shipan and Volden, 2008; Maggetti and Gilardi, 2016). They define "learning" as a process where policies in one jurisdiction are influenced by the consequences of similar policies in other jurisdictions; policy adoption is more likely if the policy has been successful elsewhere (Meseguer, 2010; Volden, 2016).[7] Political actors are routinely assumed to learn about the success or failure of a policy in other states, but behave in a "boundedly rational" way, relying on heuristics or behavioural rules rather than on optimization; adopting best practices is one of such heuristics. Empirical evidence that governments learn from each other in the sense just described is, for example, provided in Meseguer (2010), Volden (2006) or Jensen and Lindstädt (2012). Policy diffusion has been observed in a wide range of policy areas, ranging from codes of good governance to environmental policy instruments to social security reforms (Graham et al., 2013). Examples from tax policy include Leiser (2015), Arel-Bundock and Parinandi (2017) or Jensen and Lindstädt (2012).[8] Theoretical analyses are surprisingly scarce, however. Volden et al. (2008) study policy diffusion when policy options binary. In Glick (2014), social learning can resolve uncertainty about policy outcomes; tax mimicking then indicates a tendency towards efficiency. In Becker and Davies (2015), governments gradually learn about the true elasticity of their tax base; they play a best-response game. By observing its own and its neighboring countries' outcomes, a

[7]In an experimental study, Tyran and Sausgruber (2005) show that providing political decision-makers with information about policy innovations in other jurisdictions speeds up the diffusion of (efficiency-enhancing) policies.

[8]Li (2002) argues that the concept of formulary allocation of a global tax base itself "evolves" from the historical development of the arm's-length principle. However, "evolutionary" here refers to non-sudden, incremental changes to the existing system, and is only loosely related to learning and imitation.

country updates its belief function; eventually Bayesian updating leads to full knowledge, and the classical inefficiencies of tax competition will prevail. Our paper differs from these studies by focusing on imitation under a behavioural rule, using a non-binary (continuous) policy space and not using a knowledge-based interpretation of learning. Under this premise, we predict outcomes to be worse than in tax competition.

The rest of this paper is organized as follows: Section 2 sets up a general model of interacting governments that apportion a common and endogenous tax base among themselves. Section 3 presents our main results. Section 4 spells out the general model of Section 2 for a specific setting with international investments in the spirit of the CCCTB approach. In Section 5 we change government objectives from tax revenue maximization to welfare maximization and show that our principal insights fully carry over. Section 6 concludes. Proofs are relegated to the Appendix.

2 Formulary apportionment of a common tax base

2.1 Set-up

Our main points can be made in a generic setting of fiscal interactions between a set of (national) governments that shares a common and aggregate tax base (say, the consolidated taxable profits of multinational groups active in their jurisdictions). Each government can only tax a fraction of this tax base, and fractions are determined by a formula-based apportionment method. Both the size of the tax base and the assigned shares vary with governments' policy choices (i.e., tax rates).

In Section 4 we provide a full-fledged model of profit taxation with formula apportionment. Here, we present its core of taxing a common-pool resource of endogenous value and with endogenous division. This set-up captures the *reduced* form for a variety of institutionally and economically richer environments where fiscal outcomes can be traced back to the tax-affected choices and business activities of multinational firms (or other taxpayers). We look at fiscal interaction from the perspective of its players (i.e., governments) and model payoffs directly as functions of their policy instruments.

As a policy objective, we assume that governments only care for their (national) tax revenues out of the common tax base. This Leviathan perspective is less restrictive than it appears: as long as governments' objectives are related to tax revenues, our main insights fully carry over (see Section 5 for welfare maximization).[9]

We consider a common market with $n \geq 2$ identical small open economies, called countries and variably indexed by $i, j, k = 1, \ldots, n$. We aggregate the corporate sector into a single representative multinational group that operates firms in all countries in the common market.

The multinational group has to pay profit taxes to the governments of all countries where it operates. The countries operate with a commonly agreed and consolidated base, Φ, for taxing profits. We will henceforth assume that the multinational's taxable profits are always positive: $\Phi > 0$.[10]

The tax base Φ is shared between the nn countries following a method of formulary apportionment (FA). A certain fraction of the consolidated tax base Φ is assigned to each country. These fractions are meant to reflect how the multinational firm allocates its activities across countries. In a more complete model, these activities could, e.g., be represented by the share of its total capital that the multinational invests in the countries or by some other business-related factors (payroll, sales, etc.). Generally, we denote by $\alpha_i \in [0, 1]$ the (endogenous) share of tax revenues assigned to country i; the vector of shares is $\alpha = (\alpha_1, \ldots, \alpha_n)$. Shares invariably add up to unity across countries: in any circumstance,

$$\sum_{j=1}^{n} \alpha_j = 1.$$

The statutory profit tax rate in country i is denoted by t_i; we assume that for all i, $t_i \in [0, \bar{t}]$ for some $\bar{t} < 1$; these keeps strategy spaces compact and precludes expropriation.

[9]In the context of the CCCTB in the EU, the rather parochial debate among policy makers almost exclusively focus(ed) on the possible consequences for the tax proceeds in their country; all other benefits and costs of the proposal are largely ignored.

[10]Losses and loss-offsets are a thorny issue for formulary apportionment, which we choose to avoid here. See, e.g., Gérard and Weiner (2006) or Mardan and Stimmelmayr (2017).

We collect tax rates in vectors $\mathbf{t} = (t_1, \ldots, t_n)$. To single out one country, we shall sometimes write $\mathbf{t} = (t_i; \mathbf{t}_{-i})$. With FA, the tax revenues in country i amount to

$$T_i = t_i \cdot \alpha_i \cdot \Phi. \tag{1}$$

Denote by

$$\tau := \sum_i t_i \cdot \alpha_i$$

the effective tax rate on the firm's worldwide profits; tax payments (and thus tax proceeds) total $\tau\Phi$.

Taxable profits and the distribution of apportionment shares depend on the firm's choices (investments, hiring, sales etc.). At this point, we do not need to introduce any specific model of corporate decision-making (in Section 4 we will consider maximization of after-tax profits).[11] We assume, however, that in its choices the firm takes into account tax rates and tax rules. While from the firm's perspective statutory tax rates $\mathbf{t}$ are parametrically fixed, the effective tax rate τ is endogenous to the firm's decisions since the apportionment shares α_i depend on the firm's activities.

Given an apportionment formula and a definition for taxable profits, the firm's choices and, consequently, gross profits and taxable profits will vary with tax rates, $\mathbf{t}$. We write the resulting volume of taxable profits, Φ, and of the apportionment shares, α_i, as functions that depend (only) on $\mathbf{t}$:

$$\Phi = \Phi(\mathbf{t}), \quad \text{and} \quad \alpha_i = \alpha_i(\mathbf{t}) \quad \text{for } i = 1, \ldots, n.$$

As countries are identical and do not differ as locations except, possibly, in their tax rates, the following symmetry properties for profits and apportionment shares appear natural:

- if $\mathbf{t}'$ is a permutation of $\mathbf{t}$, then $\Phi(\mathbf{t}) = \Phi(\mathbf{t}')$;
- if $\mathbf{t}'_{-i}$ is a permutation of $\mathbf{t}_{-i}$, then $\alpha_i(t_i; \mathbf{t}_{-i}) = \alpha_i(t_i; \mathbf{t}'_{-i})$ for all i, t_i, t_{-i};

[11] We assume a centralized multinational group, where one management makes the decision for all international divisions of the firm.

- if $t_i = t_j$, then $\alpha_i(t_i; \mathbf{t}_{-i}) = \alpha_j(t_j; \mathbf{t}_{-j})$.

Thus, for the firm, it is only relevant *which* tax rates are set, but not by whom. Similarly, from any country's perspective, the identities of the other countries do not matter for its tax share; again it is only relevant which tax rates are set, but not by whom: relabeling the other countries would not change the value of the apportionment factor, and countries with the same tax rate are assigned identical base shares. These symmetry properties reflect an equal treatment of countries both in the definition of the consolidated tax base and the apportionment method.

In what follows we will often consider symmetric situations, i.e., situations where all countries set identical tax rates (in evolutionary parlance: "monomorphic states"): $t_i = t$ in all i. With $\mathbf{1}^n = (1, \ldots, 1)$ as the vector of n ones, such tax situations will be written as $\mathbf{t} = t \cdot \mathbf{1}^n$. In a symmetric situation the tax base is shared equally between countries: $\alpha_i(t \cdot \mathbf{1}^n) = 1/n$ for all t. Moreover, as the apportionment shares add up to unity across countries,

$$\frac{\partial \alpha_i(t \cdot \mathbf{1}^n)}{\partial t_i} = -(n-1)\frac{\partial \alpha_j(t \cdot \mathbf{1}^n)}{\partial t_i} \tag{2}$$

holds for all $j \neq i$.

For our reduced form approach, we do not make any monotonicity assumptions on functions $\Phi(\mathbf{t})$ and $\alpha_i(\mathbf{t})$. Our analysis allows taxable profits and apportionment shares to move into any direction when (domestic or foreign) tax rates vary. In fact, previous research has shown that, depending on the model of the multinational firm, many things can happen with formula apportionment.

2.2 Payoffs and solutions

Tax revenues in each country depend, via the firm's optimal choices, on the entire tax vector:

$$T_i(\mathbf{t}) = T(t_i; \mathbf{t}_{-i}) = t_i \cdot \alpha_i(t_i; \mathbf{t}_{-i}) \cdot \Phi(\mathbf{t}).$$

By our previous assumptions, tax revenues are symmetric functions: $T_i(t_i; \mathbf{t}_{-i}) = T_i(t_i; \mathbf{t}'_{-i})$ if $\mathbf{t}'_{-i}$ is a permutation of $\mathbf{t}_{-i}$. Assuming that they all exist and are unique, we consider the following solutions:

Definition 1 *Suppose that a method* $(\alpha_1(\mathbf{t}), \ldots, \alpha_n(\mathbf{t}))$ *of formulary apportionment is applied to a well-defined consolidated tax base* $\Phi(\mathbf{t})$. *Then:*

- *A symmetric cooperative solution is a tax rate* t^* *that, when applied everywhere, maximizes joint tax revenues:*

$$\sum_j T_j(t^* \cdot \mathbf{1}^n) \geq \sum_j T_j(t \cdot \mathbf{1}^n) \quad \text{for all } t.$$

- *A tax rate* t^N *is played in a symmetric Nash equilibrium of a non-cooperative tax competition game if*

$$T_i(t^N \cdot \mathbf{1}^n) \geq T_i(t; t^N \cdot \mathbf{1}^{n-1}) \quad \text{for all } t, i.$$

- *A tax rate* t^E *is said to be an evolutionarily stable strategy (ESS) if*

$$T_j(t^E; t, t^E \cdot \mathbf{1}^{n-2}) \geq T_i(t; t^E \cdot \mathbf{1}^{n-1}) \quad \text{for all } t, i, j.$$

In a cooperative solution, the total of tax revenues is maximized. In a Nash equilibrium, no country can earn higher tax revenues from a unilateral deviation, given the tax choices of the other countries. At an evolutionarily stable profile no country can gain a strict relative advantage over any other country from deviating; the payoff comparison is between the (single) deviator, who chooses policy t, and the non-deviators, who all stick to t^E. An ESS can be understood as the Nash equilibrium when governments care about their *relative* performance (Schaffer, 1988). Formally, an ESS then is a strategy t^E such that

$$t^E = \arg\max_t \left[T_i(t; t^E, \ldots, t^E) - T_j(t^E; t, t^E, \ldots, t^E) \right]. \tag{3}$$

If the ESS is unique and interior, it is determined by the following FOCs:

$$\frac{\partial(T_i - T_j)}{\partial t_i} = \frac{\partial}{\partial t_i}\left[T_i(t; t^E \cdot \mathbf{1}^{n-1}) - T_j(t^E; t, t^E \cdot \mathbf{1}^{n-2})\right]\Big|_{t=t^E} = 0$$

for all i, j (Tanaka, 2000). A finite-population ESS generally is neither a cooperative solution nor a Nash equilibrium strategy of the "absolute" game (Schaffer, 1988; Hehenkamp et al., 2010). Deviating from a Nash equilibrium may pay off in relative terms even if it reduces absolute payoffs. Sano (2012), Wagener (2013) or Philipowski (2015) show this for standard tax competition games.

The ESS is a static, one-shot concept. Recall, however, that when the interaction between government occurs repeatedly over a long horizon and takes the form of imitation-cum-experimentation dynamics, the evolutionary stability of a strategy in the stage game is a necessary condition for that strategy to be a rest point (technically, a stochastically stable state) of the imitation dynamics (Alós-Ferrer and Schlag, 2009). In models of fiscal interaction, the ESS is, thus, the theoretically predicted outcome of laboratory federalism with its learning and innovation processes.

3 General results

3.1 Characterization

Define by

$$\varepsilon_\Phi(t) := -\frac{t}{\Phi(t \cdot \mathbf{1}^n)} \cdot \frac{\partial \Phi(t \cdot \mathbf{1}^n)}{\partial t_i} \quad \text{and} \quad \varepsilon_\alpha(t) := -nt \cdot \frac{\partial \alpha_i(t \cdot \mathbf{1}^n)}{\partial t_i}$$

the elasticities of, respectively, the tax base and the apportionment share with respect to a tax rate, evaluated at a symmetric situation (recall that $\alpha_i = 1/n$ then). Due to symmetry, these elasticities do not depend on the index i of the tax rate that changes.

For interior symmetric solutions (i.e., $t_i = t \in (0, \bar{t})$ for all i) in the reduced form of tax competition under FA we get the following result:

Proposition 1 • *An interior symmetric cooperative solution t^* is characterized by*

$$\varepsilon_\Phi(t^*) = \frac{1}{n}. \tag{4}$$

• *An interior symmetric Nash equilibrium t^N is characterized by*

$$\varepsilon_\Phi(t^N) + \varepsilon_\alpha(t^N) = 1. \tag{5}$$

• *An interior ESS t^E is characterized by*

$$\varepsilon_\alpha(t^E) = 1 - \frac{1}{n}. \tag{6}$$

Condition (4) in the first item of Proposition 1 is the standard condition on the tax elasticity of the tax base for the maximum of a Laffer curve.[12] Condition (5) in the second item conveys that in a Nash equilibrium each government balances the direct revenue effects of a change in its tax rate (on the LHS) against a *formula effect*, encompassing that a tax change also affects the fraction of corporate profits assigned to the country. The ESS condition (6) in the third item deserves closer inspection. It is, evaluated in a symmetric situation, the first-order condition for maximizing $(\alpha_i t_i - \alpha_j t_j)$, i.e., the condition to maximize the gap between country i's and country j's tax claim on the common consolidated tax base. This reflects the relative payoff concerns inherent in an ESS: each country strives to get ahead of the others as far as possible – and this works via appropriating a relatively larger share of the tax cake.

Condition (6) refers to an interior ESS. Obviously, it necessitates $\frac{\partial \alpha_i}{\partial t_i} < 0$, i.e., that increasing the tax rate in a country diminishes the share of the common tax base attributed to this country. Below we will discuss the economic prerequisites for such an effect. As the proof of the third item of Proposition 1 makes clear, no interior ESS can exist if the apportionment exhibits $\frac{\partial \alpha_i}{\partial t_i} \geq 0$ everywhere; then the ESS will be the *maximum* tax rate.

[12]Observe that in a symmetric situation a single tax rate t_i only impacts on the total tax rate by a factor $1/n$.

Proposition 1 conveys a number of important messages. First and evidently, the three solutions generally differ from one another: efficiency, Nash equilibrium and ESS do not coincide. Second, in reduced form, for the cooperative solution only the tax base, $\Phi(\mathbf{t})$, matters; the apportionment formula (represented by the α_i) is irrelevant. The reason is that the apportionment only determines the distribution of tax revenues; from the perspective of joint revenue maximization this sharing is, however, irrelevant. For the ESS this is exactly the other way round: only the apportionment method, represented by $\alpha_i(\mathbf{t})$, matters, the dependence of the tax base on $\mathbf{t}$ is irrelevant. The intuition is that the tax base is common to all countries and, thus, does not affect the relative positions of countries vis-à-vis each other; only the apportionment factor determines relative payoffs. The Nash equilibrium depends on both the apportionment method and the tax base definition – simply as both these elements affect national tax revenues.

It is important to note that these statements refer to *reduced* definitions of tax base and apportionment shares, understood as functions of tax rates, $\mathbf{t}$. In practice, neither Φ nor the α_i are defined in terms of tax rates but rather in terms of the firm's activities. We will return to this in Section 4.

3.2 Comparison of solutions

Given our assumption that the cooperative solution is unique, we obtain that a Nash equilibrium tax rate t^N is too high [too low], relative to t^*, if a negative [positive] fiscal externality prevails:

$$\frac{\partial T_j(t^N \cdot \mathbf{1}^n)}{\partial t_i} \leq 0 \quad \text{for } i \neq j \iff \quad t^N \geq t^*. \tag{7}$$

Due to symmetry, this condition will be simultaneously met or violated in all countries.

Proposition 2 *At a symmetric Nash equilibrium t^N a positive fiscal externality prevails (i.e., $t^N < t^*$) if*

$$\varepsilon_\alpha(t^N) > 1 - \frac{1}{n}. \tag{8}$$

Proposition 2 reiterates the general insight that tax competition under formula apportionment can cause both races over the top and to the bottom, depending on how the apportionment method and the common tax base are designed (Pethig and Wagener, 2007; Eichner and Runkel, 2012).[13]

Equations (8) and (6) reveal an interesting feature: the condition that fiscal externalities are zero is identical to the FOC for an ESS. Hence,

Proposition 3 *If the Nash equilibrium coincides with the cooperative outcome under formulary apportionment, then it is an ESS: if* $t^N = t^*$*, then* $t^E = t^N$*.*

Technically, this striking result comes from a general observation in Hehenkamp et al. (2010, Corollary 2): for all symmetric games with differentiable payoffs T_i and compact strategy sets, equality of Nash equilibrium and ESS ($t^N = t^E$) necessitates that $\partial T_j(t^N \cdot \mathbf{1}^n)/\partial t_i = 0$ for all $j \neq i$. Since a symmetric Nash equilibrium itself satisfies $\partial T_i(t^N \cdot \mathbf{1}^n)/\partial t_i = 0$ for all i, identity of Nash equilibrium and ESS implies the cooperative outcome: $\partial/\partial t_i \left(\sum_k T_k(t^N \cdot \mathbf{1}^n)\right) = 0$.

Proposition 3 conveys that an apportionment method and a tax base definition that together internalize fiscal externalities in Nash play also render the ESS efficient. Hence, if the social planner manages to construct an efficient apportionment mechanism (in the sense that best-response play leads to a cooperative outcome), imitation-cum-experimentation dynamics would, in the long run, also implement cooperative outcomes. Such efficient mechanisms do indeed exist (see, e.g., Pethig and Wagener, 2007, Prop. 4), but they are knife-edge cases.

If the cooperative outcome is not known (neither to the FA designer nor to national governments), fiscal externalities are likely to remain non-internalized. In these cases, the three solutions can be ranked:

[13] Kolmar and Wagener (2007) show that tax competition leads to inefficiently low tax rates if and only if the investment elasticity of the tax base is lower than the investment elasticity for the apportionment factor. This fully accords with the results to come.

Proposition 4 *In the presence of positive revenue externalities (i.e., if* $\frac{\partial T_j(\mathbf{t})}{\partial t_i} > 0$ *for all* $\mathbf{t}$ *and* $i \neq j$*),*

$$t^E < t^N < t^*.$$

In the presence of negative revenue externalities, this ranking reverses.

Proposition 4 states that evolutionary play amplifies the inefficiencies that already prevail under Nash play: both races over the top and to the bottom are sped up. The intuition is straightforward (see, e.g., Wagener, 2013): under relative performance concerns, each government can increase its relative standing by increasing its own payoff and by reducing that of others. If externalities are positive [negative] the second, spiteful motive adds an incentive to further lower [increase] one's tax rate beyond what the first motive – which matters under Nash play – dictates.

4 A model with international investments

4.1 General

So far, our analysis dealt with an abstract and reduced-form economy. To flesh out our approach, this section offers a more specific model that explicitly takes into account firm behaviour in a CCCTB framework.

Suppose that the multinational group aims to maximize its worldwide profits, net of taxes. As usual, net profits are gross economic profits minus tax payments, $\tau\Phi$. Taxable profits, Φ most likely differ from gross economic profits due to non-deductible expenses, tax-depreciation rules, exemptions, allowances, valuation rules, the in- or exclusion of unrealized incomes and many other details. Such divergences generally imply the non-neutrality of profit taxation.

The multinational pursues its aim by deciding how much capital, k_i, to invest in each country $i = 1, \ldots, n$ of its operations. Other business decisions (such as hiring labor, choosing sales levels, setting prices etc.) are ignored for simplicity. Investment decisions are summarized in $\mathbf{k} = (k_1, \ldots, k_n)$; we write $\mathbf{k} = (k_i; k_{-i})$ if we need to distinguish country i from other countries. Denote total investments by

$$K := \sum_j k_j.$$

We assume that total economic gross profits of the multinational, $\tilde{\Pi}^g$, depend in a symmetric way on investments,

$$\tilde{\Pi}^g : \mathbb{R}^n_+ \to \mathbb{R} \quad \text{with} \quad \mathbf{k} \mapsto \tilde{\Pi}^g(\mathbf{k}),$$

where $\tilde{\Pi}^g(\mathbf{k}) = \tilde{\Pi}^g(\mathbf{k}')$ if $\mathbf{k}'$ is a permutation of $\mathbf{k}$. We assume that $\tilde{\Pi}^g$ takes on strictly positive values (at least) over some subset of $\mathbb{R}^n_+$. Likewise, in this expanded version of the model, we assume that taxable profits $\tilde{\Phi}$ depend on investments $\mathbf{k}$: $\tilde{\Phi} = \tilde{\Phi}(\mathbf{k})$; the relation of $\tilde{\Phi}(\mathbf{k})$ to $\Phi(\mathbf{t})$ from Section 2 will be made clear below. As with gross profits, $\tilde{\Phi}$ is assumed to be a symmetric function.

Apportionment maps the firm's activities into shares of the tax base $\tilde{\Phi}$; it is represented by symmetric functions

$$\tilde{\alpha}_i = \tilde{\alpha}(k_i; \mathbf{k}_{-i}).$$

We assume that

- apportionment shares always add up to unity: $\sum_j \tilde{\alpha}_j = 1$ for all $\mathbf{k}$;
- a country's share of the tax base increases with the amount of the investment in its jurisdiction, i.e.,

$$\frac{\partial \tilde{\alpha}(k_i; \mathbf{k}_{-i})}{\partial k_i} > 0$$

  for all $\mathbf{k}$. By symmetry and the adding-up property, this implies that $\frac{\partial \tilde{\alpha}(k_i; \mathbf{k}_{-i})}{\partial k_j} < 0$ for all $j \neq i$.

The simplest and most prominent example of an apportionment method is *property share* apportionment:

$$\tilde{\alpha}_i = k_i/K.$$

Other examples from practice such as *sales apportionment* or *payroll apportionment* show that apportionment methods often are generalized weights,

$$\tilde{\alpha}_i(\mathbf{k}) = \frac{g(k_i)}{\sum_k g(k_k)}, \tag{9}$$

where $g(k_i)$ is a suitably defined function that maps investments (as a proxy for the firm's activities in country i) into sales, payroll or whatever. For example, all components of the so-called Massachusetts formula are of this type (Gordon and Wilson, 1986).

The multinational firm maximizes, by choice of $\mathbf{k}$, its net profits,

$$\tilde{\Pi}^n(\mathbf{k}) = \tilde{\Pi}^g(\mathbf{k}) - \tau\tilde{\Phi}(\mathbf{k}),$$

where $\tau = \sum_j t_j\tilde{\alpha}_j$ as before denotes the total tax rate. We assume that $\tilde{\Pi}^n(\mathbf{k})$ is pseudo-concave in $\mathbf{k}$, such that the optimal investment behaviour satisfies the FOCs,

$$0 = \frac{\partial\tilde{\Pi}^n}{\partial k_i} = \frac{\partial\tilde{\Pi}^g}{\partial k_i} - \tau\cdot\frac{\partial\tilde{\Phi}}{\partial k_i} - \tilde{\Phi}(\mathbf{k})\cdot\frac{\partial\tau}{\partial k_i} \quad \text{for } i = 1, \ldots, n. \tag{10}$$

Here,

$$\frac{\partial\tau}{\partial k_i} = \sum_j t_j\cdot\frac{\partial\tilde{\alpha}_j}{\partial k_i}$$

captures the change in the effective tax rate, τ, which the multinational can induce via the apportionment shares. Condition (10) tells us that, with formula apportionment, taxation not only affects factor demand through its direct effect on net marginal profits (the first and second term on the RHS), but also indirectly via the apportionment method (the third term on the RHS). The firm accounts for the (positive or negative) dependence of its effective tax rate on its input choices.

The set of equations in (10) determines investments as a function of tax rates: $\mathbf{k} = \mathbf{k}(\mathbf{t})$. Plugging this into $\tilde{\Phi}$ and the $\tilde{\alpha}_i$ yields the functions $\Phi(\mathbf{t})$ and $\alpha_i(\mathbf{t})$ that we used in the previous sections. In particular, we get

$$\frac{\partial \alpha_i(\mathbf{t})}{\partial t_i} = \sum_j \frac{\partial \tilde{\alpha}_i}{\partial k_j}\frac{\partial k_j}{\partial t_i} \quad \text{and} \quad \frac{\partial \Phi(\mathbf{t})}{\partial t_i} = \sum_j \frac{\partial \tilde{\Phi}}{\partial k_j}\frac{\partial k_j}{\partial t_i}. \tag{11}$$

The multinational's responses to tax changes under formulary apportionment are quite complex in general; as previous research has shown, they can go in any direction, depending on the definition of the tax base, $\tilde{\Phi}$, and the apportionment method, $\tilde{\alpha}$. For symmetric situations, expressions for the direct $(\partial k_i(t \cdot \mathbf{1}^n)/\partial t_i)$ and the cross-border effects $(\partial k_i(t \cdot \mathbf{1}^n)/\partial t_j)$ of tax increases are provided in Eqs. (A.8) and (A.9) in Appendix A.4.

4.2 Symmetric situations

Proposition 1 highlighted the importance of the *tax* elasticities of apportionment share and tax base definition. In our more specific model, these tax elasticities can be traced back to the direct properties of apportionment method tax base and the tex responses of investments.

Let us again focus on symmetric situations where $t_i = t$ for all i. Then $\tau = t$ and $\alpha_i = 1/n$. From (10) and the symmetry of $\tilde{\Phi}$ and $\tilde{\Pi}^g$, investment levels are the same everywhere $(k_i = k_j)$ and comparative statics are also symmetric:

$$\frac{\partial k_i(t \cdot \mathbf{1}^n)}{\partial t_i} = \frac{\partial k_j(t \cdot \mathbf{1}^n)}{\partial t_j} \quad \text{and} \quad \frac{\partial k_i(t \cdot \mathbf{1}^n)}{\partial t_j} = \frac{\partial k_j(t \cdot \mathbf{1}^n)}{\partial t_i} \quad \text{for all}\, i \neq j.$$

Consequently, in (11):

$$\begin{aligned} \frac{\partial \alpha_i(t \cdot \mathbf{1}^n)}{\partial t_i} &= \frac{\partial \tilde{\alpha}_i(k \cdot \mathbf{1}^n)}{\partial k_i}\frac{\partial k_i}{\partial t_i} + (n-1)\frac{\partial \tilde{\alpha}_k(k \cdot \mathbf{1}^n)}{\partial k_j}\frac{\partial k_j}{\partial t_i} \\ &= \frac{\partial \tilde{\alpha}_i(k \cdot \mathbf{1}^n)}{\partial k_i} \cdot \frac{\partial}{\partial t_i}\left(k_i(t \cdot \mathbf{1}^n) - k_j(t \cdot \mathbf{1}^n)\right) & (12) \\ \frac{\partial \Phi(t \cdot \mathbf{1}^n)}{\partial t_i} &= \frac{\partial \tilde{\Phi}(k \cdot \mathbf{1}^n)}{\partial k_i} \cdot \frac{\partial K(t \cdot \mathbf{1}^n)}{\partial t_i}. & (13) \end{aligned}$$

Equation (12) shows that the tax effects on the apportionment share are shaped by the behaviour of investment *differentials*, $k_i - k_j$: an increase in the tax rate of a country reduces that country's apportionment share if and only if the tax effect on domestic investment is smaller than on investment in a foreign country, $\partial k_i/\partial t_i < \partial k_j/\partial t_j$. An economically plausible case would be that a tax increase in i drives out capital from that country and into the other countries $j \neq i$, i.e., $\partial k_i/\partial t_i < 0 < \partial k_j/\partial t_j$.

Equation (13) conveys that an increase in any tax rate increases [reduces] total taxable profits if and only if total investments, K, increase [diminish] in response to the tax hike. This will matter both for the characterization of Nash equilibria and efficient outcomes below.

4.3 Evolutionary stability

Exploiting the comparative statics of investments with respect to tax rates (see Appendix A.4), we can obtain a direct characterization of an ESS. Define

$$D := \left(\frac{\partial^2 \tilde{\Pi}^n}{\partial k_i^2} - \frac{\partial^2 \tilde{\Pi}^n}{\partial k_i \partial k_j} \right) < 0,$$

which, as we show in Appendix A.4, is closely related to the tax elasticity of investment and, thus, measures capital mobility.[14]

Proposition 5 *An interior ESS t^E under formulary apportionment is characterized by*

$$1 = -\frac{t^E \tilde{\Phi}}{D} \frac{n^3}{(n-1)^2} \left(\frac{\partial \tilde{\alpha}_i(k\mathbf{1}^n)}{\partial k_i} \right)^2. \tag{14}$$

To understand why the square of $\partial \tilde{\alpha}_i/\partial k_i$ shows up in Proposition 5, recall (12): here, the change in the relative position for investments, $k_i - k_j$, is determined by $\partial \tilde{\alpha}_i/\partial k_i$ (see (A.10) in Appendix A.4). The apportionment method, thus, exerts a double impact: one direct, via the tax base share, and one indirect, via the investment differential.

[14]For general, non-symmetric situations, D is defined in (A.7) in Appendix A.4; D denotes the determinant of the Jacobian inverse in the comparative statics of capital investments. In symmetric situations, it boils down to D, as given here.

The fraction $n^3/(n-1)^2$ in (14) might irritate at first sight. Consider, however, apportionment formulas of the weight-type (9). Here,

$$\frac{\partial \tilde{\alpha}_i(k\mathbf{1}^n)}{\partial k_i} = \frac{1}{\left(\sum_j g(k_j)\right)^2} \sum_{j \neq i} g(k_j) g'(k_j) \Bigg|_{\mathbf{k}=k\mathbf{1}^n} = \frac{n-1}{n^2} \frac{g'(k)}{g(k)}. \tag{15}$$

In such cases, (14) boils down to

$$1 = -\frac{t^E \tilde{\Phi}}{nD} \left(\frac{g'(k)}{g(k)}\right)^2. \tag{16}$$

From (16), the ESS tax rate is higher the less responsive the apportionment formula (as proxied by g'/g) and the less elastic is capital demand (as proxied by $-D^{-1}$). These factors also shape Nash play (see, e.g., Pethig and Wagener, 2007, Section 3.2); both for relative and absolute payoff concerns, they reflect that capital tax competition is stiffer the more elastic the tax base is with respect to tax changes.

From Proposition 1 and Eqs. (12) and (13), similar expressions as (14) can also be derived for efficient outcomes and Nash equilibrium. Given the familiarity of these concepts, we refrain from discussing them here.

4.4 Separable profits

In much of the literature on FA both economic and taxable profits are assumed to be additively separable between countries. I.e., there exist functions π and ϕ such that

$$\tilde{\Pi}^g(\mathbf{k}) = \sum_j \pi(k_j) \quad \text{and} \quad \tilde{\Phi}(\mathbf{k}) = \sum_j \phi(k_j), \tag{17}$$

where $\pi(k_i)$ and $\phi(k_i)$ measure, respectively, economic and taxable profits in country i, and consolidation simply boils down to adding up. With (17), net profits amount to $\tilde{\Pi}^n = \sum_j \left(\pi(k_j) - \tau\phi(k_j)\right)$. In a symmetric situation with tax rate t everywhere, profit-maximizing investment (10) in each country is given through

$$\pi'(k) - t\phi'(k) = 0. \tag{18}$$

The SOC requires that

$$D(k) := \pi''(k) - t\phi''(k)$$

is negative. We then obtain the following specification of Proposition 1:[15]

Proposition 6 *Suppose that profits are separable and the apportionment method is a weighing scheme (9). The tax rate and the attending level of investments are jointly characterized by (18) and, ...*

- *in an interior efficient solution t^*, by*

$$0 = \phi(k) + \frac{t}{D(k)} \left(\frac{\partial \phi(k)}{\partial k} \right)^2 ; \tag{19}$$

- *in an interior Nash equilibrium t^N, by*

$$0 = \phi(k) + \frac{t}{nD(k)} \left[\left(\frac{\partial \phi(k)}{\partial k} \right)^2 + (n-1)\phi^2(k) \left(\frac{g'(k)}{g(k)} \right)^2 \right] ; \tag{20}$$

- *in an interior ESS t^E, by*

$$1 = -\frac{t}{D(k)} \phi(k) \left(\frac{g'(k)}{g(k)} \right)^2 . \tag{21}$$

The efficient solution and the ESS are independent of the number of countries, n.

A striking observation in the third item of Proposition 6 is the independence of the ESS of n:[16] irrespective of the number of countries that will share the consolidated tax base,

[15] The square of $\frac{\partial \phi(k)}{\partial k}$ in (19) and (20) can be explained in a similar way as the square of $\frac{\partial \tilde{\alpha}_i}{\partial k_i}$ in (14). As shown in (A.11), the tax responsiveness of total investments K depends on the investment responsiveness of the tax base, $\tilde{\Phi}$. Hence, the latter shows up squared in (13).

[16] Consisting of only two equations each, Proposition 6 provides a convenient recipe to calculate efficient outcomes, Nash equilibrium, and ESS.

the outcome under evolutionary play will always be the same. This is in marked contrast to the Nash equilibrium (see second item in Proposition 6), which generally varies with n.

The independence of the ESS with respect to the number of players is due to a feature of evolutionary play that, in large classes of games, it leads to "perfectly competitive" outcomes (Alós-Ferrer and Ania, 2005): each of the finitely many players behave as if there were an infinity of players none of which had any impact on the "aggregates" of the economy.[17] In the separable case, this also holds for tax competition with a CCCTB; it is currently an open question whether it also holds in the general, non-separable case.

Clearly, Proposition 6 confirms Proposition 4: if any two solutions coincide, then so does the third. To illustrate Proposition 3, we now provide a worked example for the separable case.

4.5 A worked example

Suppose that production in each jurisdiction follows a quadratic production function,

$$f(k) = (B - bk)k,$$

with $B, b > 0$; we shall make sure that marginal productivity, $B - 2bk$, is positive at any situation that we consider. Assume that the sales price of the multinational's output equals one worldwide and that sales equal production. Suppose that the actual costs of capital are constant and equal $r > 0$ per unit, and that $h \cdot r$ (with $h \geq 0$) are the capital costs that are tax-deductible in profit taxation. Hence, economic and taxable profits in a country are given by, respectively,

$$\pi^g(k) = f(k) - rk \quad \text{and} \quad \phi(k) = f(k) - h \cdot rk.$$

[17]To see that the ESS corresponds to the competitive outcome verify that the ESS condition (21) coincides with the Nash condition (20) for $n \to \infty$.

Obviously, $h < 1$ indicates a situation where costs of capital are less than fully tax-deductible, $h > 1$ would imply a tax-subsidy on capital costs, and $h = 1$ corresponds to "pure" profit taxation.

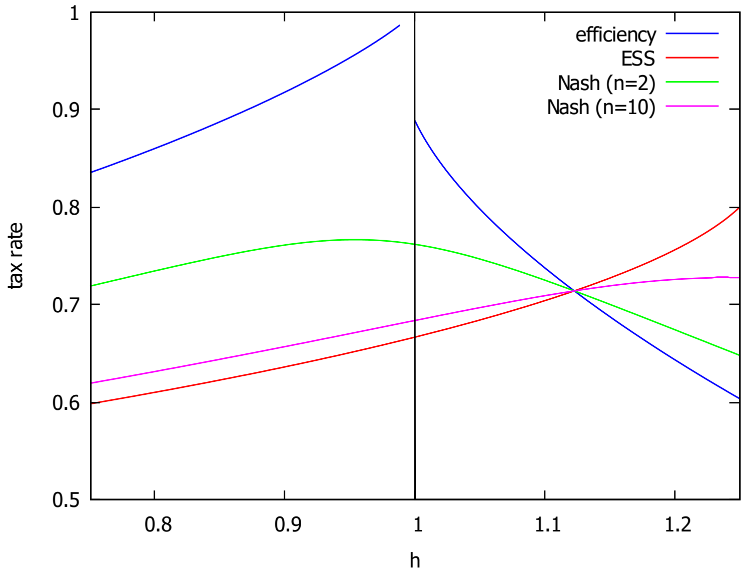
In a symmetric situation with tax rate t everywhere, net-of-tax profit maximization (18) leads to

$$k(t) = \frac{1}{2b}\left(B - r\frac{1 - h \cdot t}{1 - t}\right), \tag{22}$$

which decreases [increases] with the tax rate whenever $h < 1$ [resp., $h > 1$]. We assume property-share apportionment: $\alpha_i = k_i/K$.

Figure 1 depicts, for varying definitions of the tax base (captured by h), the tax rates at an efficient solution, the ESS, and the Nash equilibria for $n = 2$ and $n = 10$ countries.[18]

Figure 1: Different solutions with CCCTB ($B = 5$, $b = r = 1$)



Ignoring the case $h = 1$ for the moment, Figure 1 portrays the following messages:

- There exists a situation (at roughly $h = 1.12$) where efficient outcome, ESS, and Nash equilibrium (for any n) coincide. This is the "ideal" situation characterized in Proposition 3.

[18]Tax rates were calculated from Proposition 6, using the computer algebra system Maxima. The Maxima source code, which also generates Figure 1, is available from the author.

- At levels of cost deductibility below this ideal situation ($h < 1.12$), we observe $t^* > t^N(n) > t^E$ for all n. In line with Proposition 4, the ESS amplifies the under-taxation that prevails in a Nash equilibrium. Moreover, if $n' > n$, then $t^N(n') < t^N(n)$ – "standard" tax competition intensifies if the number of countries increases. By contrast, the ESS is independent of n.

- At values of h above the ideal situation ($h > 1.12$), we see that $t^E > t^N(n) > t^*$ for all n. The ESS now amplifies the *over*-taxation in a Nash equilibrium, which again is in line with Proposition 4. Again, the ESS is independent of n, unlike the Nash equilibrium tax rate which increases with the number of countries, n.

Finally, let us consider the knife-edge case $h = 1$. Here, investments (22) are independent of the tax rate, t. Hence, taxation is essentially lump-sum, and we have $t^* = t^N = t^E = 1$ (or the largest possible tax rate, $\bar{t}$). Figure 1 fails to depict this for Nash equilibria and ESS; the graphs should exhibit a discontinuity here. Observe that Figure 1 does not contradict any of our results, which are carefully conditioned on *interior* solutions.

5 Welfare maximization

Our analysis has so far assumed that governments aim at maximizing tax revenues. Proposition 4 showed that ESS, Nash equilibrium, and efficient outcomes can be ranked. Here we will illustrate that and how our main insights can be transferred to alternative settings, at least as long as governments pursue objectives that are somehow related to tax revenues.[19] To illustrate this point, let us suppose that governments care for the well-being of a representative citizen in their countries. The approach is inspired by Pinto (2007), Wrede (2014) or Matsumoto (2016).

[19]Nielsen et al. (2010) show that a comparison of Nash-play tax competition under separate accounting and under property-share FA basically yields the same results for both revenue- and for welfare-maximizing governments.

A citizen in country i derives utility from private consumption, c_i, and from a government-provided good, g_i. Assuming identical preferences in all countries, the utility of a citizen of state i is represented by

$$U_i = u(c_i, g_i),$$

which we assume to be strictly increasing in both arguments, twice differentiable, strictly quasi-concave and to satisfy the Inada conditions. Individuals finance their private consumption from the net profits of the multinational enterprise, while public consumption is financed by national revenues from the corporate income tax. Assuming, for symmetry, that citizens in country i own a share of $1/n$ of the multinational, private consumption in i thus amounts to

$$c_i = \frac{\Pi^n}{n}.$$

Public consumption amounts to

$$g_i = T_i = t_i \alpha_i \Phi.$$

Following the reduced-form approach of Section 2 and imposing the same symmetry assumptions, we directly write utilities as functions of the tax vector. Plugging all relevant components into the social welfare function then gives governments' payoffs as

$$U_i = U(t_i; \mathbf{t}_{-i}) \quad := \quad u\left(\frac{1}{n}\Pi^n(\mathbf{t}),\, t_i \alpha(t_i; \mathbf{t}_{-i}) \Phi(\mathbf{t})\right).$$

Exchanging the T_j by U_j in Definition 1, we can now define symmetric efficient outcome, symmetric Nash equilibrium and ESS for the welfare game. We then obtain

Proposition 7 *Suppose that the symmetric efficient outcome, the symmetric Nash equilibrium and the ESS are unique in the welfare game with formulary apportionment.*

- *If $t^N \leq t^*$, then $t^E \leq t^N$. Conversely, if $t^N \geq t^*$, then also $t^E \geq t^N$.*
- *$t^N \leq t^*$ holds if and only if a tax increase in i worsens the relative fiscal position of country i, i.e., if the tax revenue differential with respect to any other country*

marginally decreases: $\frac{\partial(T_i(\mathbf{t}^N)-T_j(\mathbf{t}^n))}{\partial t_i} \leq 0$.

The first item of Proposition 7 conveys that evolutionary play amplifies the inefficiencies of Nash play: both races over the top and to the bottom speed up. This corroborates the observations of Propositions 3 and 4. The intuition is the same as before: evolutionary play involves aggregate-taking behaviour. The "aggregate" here is private consumption, c, which – by the symmetry of the ownership distribution of the multinational – is the same across countries and, thus, does not matter in relative comparisons. Welfare differentials between countries can therefore only result from public goods, g_i, which are proportional to tax revenues – this is the message of the second item of Proposition 7. For tax revenues, Proposition 4 has already ranked outcomes. Echoing Proposition 3, Proposition 7 implies that equality of Nash equilibrium and ESS is tantamount to efficiency.

6 Discussion and conclusions

Being unprecedented, implementing formulary apportionment in the EU would be a courageous step in the sense that its outcome is hard to predict in advance. Beyond the EU project, lack of knowledge and the complexity and multitude of spillovers under any system of taxation with formulary apportionment might preclude that national tax policies follow an optimization approach with best response-type of play among countries. Policy mimicking and occasional innovations, however, remain a viable, even if boundedly rational way of policy-making.

Together with theoretical results on learning dynamics, this motivates why we study evolutionary stability of tax competition under formulary apportionment. Our results suggest the following: innovation-and-imitation dynamics generally do not lead to an efficient outcome. Rather, they lead further away from efficiency than uncoordinated best-response play. The direction of this divergence is determined by the sign of fiscal externalities (i.e., the cross-country revenue effects of tax changes) – on which relatively much is known from empirical and theoretical studies on formulary apportionment under the assumptions of best-response play and full knowledge.

As a first attempt, our analysis rests on a number of critical assumptions. For conclusion, let us discuss some of them – and the possibilities and consequences of giving them up.

Institutional richness and tax laws. Our models both in Sections 2 and 4 are institutionally quite abstract. Apportionment formulas and definitions of economic and taxable profits remained largely unmodelled. Our principal insights hold at this level of generality. Recent research on the CCCTB has shown, however, that more specific predictions (e.g., on whether positive or negative fiscal externalities prevail or on the size of the gap between efficiency, Nash equilibrium and ESS) crucially depend on details in the definitions of taxable profits and of the factors in the apportionment formula. In the definition of taxable profits, examples would include the treatment of corporate equity, (super-)deductions for certain expenditure categories, exempt revenues etc.; in the apportionment factor, the treatment of intangibles, the delineation of domestic capital, payroll and sales might matter.

Water's edge. The CCCTB in the recent EU proposal (European Commission, 2016) will only apply within the EU, implying that traditional transfer pricing and permanent establishment concepts will still be applied at the "water's edge", where EU firms transact with their affiliates outside the EU. In practical terms, this makes it questionable how effective the CCCTB will be at reducing tax dodging, compared to the *status quo* plus nowadays BEPS (Riedel and Runkel, 2007). From this paper's viewpoint of learning among governments, water's edge issues relate to the question of whom precisely players (governments) include in their set of comparables, when assessing and revising their policies. Alós-Ferrer and Weidenholzer (2008) argue that the distinction between the interaction environment (with whom is one economically connected?) and the information environment (whom does one observe?) is crucial for the outcome of learning processes.

Coordinated deviations. The concept of ESS entails stability of a tax policy against single-player experiments. In general, evolutionary analysis focuses on symmetric situations where some policy t has spread out to the entire player population, and then considers strategy profiles of the form

$$\mathbf{t} = (t', \overset{m}{\ldots}, t', t, \overset{n-m}{\ldots}, t),$$

where a number m of players deviate to some other t' while the other $n-m$ players stick to the initial t. The initial strategy t is stable against mutations by m players, if for no simultaneous m-fold mutation to the payoff of a mutant exceeds the payoff of a non-mutant (evolutionary stability is the case $m = 1$). If a tax policy resists any number $1 \leq m \neq n-1$ of mutations, then it is called globally stable. Global stability is a very stark property, implying that no simultaneous policy experiment can uproot a strategy, however large the group of experimenters is. Multiple deviations might be relevant for the EU when groups of member states coordinate their policies. All results in this paper do, however, hold for any number of mutations; and the reader can replace "evolutionarily stable" by "globally stable" in all our results.

Learning and knowledge. The evolutionary stability of fiscal interactions is conceptually inspired by the idea of laboratory federalism and its imitation-with-experimentation processes. These processes can be modified in various ways (Alós-Ferrer and Schlag, 2009); as long as they entail that the most successful policies will always be copied with positive probability by other jurisdictions, a policy vector being an ESS [globally stable] is a necessary requirement for being immune against rare [any number of simultaneous] policy experiments. Hence, if a unique ESS exists, the dynamics need not be spelt out. Otherwise, however, an explicit dynamic analysis is needed. The same holds for alternative learning processes (with memory or frequency-based imitation, say).

Our model does not make any specific assumptions on *what* governments actually know. Also the learning process is not defined in any epistemological terms but – in line with the policy diffusion literature – as the act of comparing payoffs and policy imitation. Clearly, in the course of time governments can be expected to update their priors (however ignorant they might have been initially) and utilize their state of knowledge in decision making. Our model could capture this by the way governments choose their policy experiments. In the imitation dynamics, information beyond payoffs and policies does not play any role. As Becker and Davies (2015) argue, (Bayesian) updating will eventually lead to perfect knowledge, allowing for best-response play with deterministic payoffs. Combining epistemological and behavioural approaches towards learning might be a realistic and interesting compromise.

Asymmetries. A clearly unrealistic assumption in our paper is that countries are identical. The assumption is not easily replaced, however. First, tax competition with formulary apportionment and asymmetric players is notoriously complex to analyze in itself (Wrede, 2014).[20] Moreover, imitative learning among asymmetric agents is a conceptually tricky issue (Alós-Ferrer and Schlag, 2009). In laboratory federalism, only few approaches deal with non-identical jurisdictions (see, e.g., Ania and Wagener, 2014; Philipowski, 2015), although in quite specific settings. Clearly, there is much scope for future research here.

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Appendix: Proofs

A.1 Proof of Proposition 1

- Cooperative solution: The FOC is given by

$$\frac{\partial}{\partial t_i}\sum_k T_k = \frac{\partial}{\partial t_i}(\tau\Phi) = \tau\frac{\partial \Phi}{\partial t_i} + \Phi\left(\alpha_i + \sum_k t_k \frac{\partial \alpha_k}{\partial t_i}\right) = 0.$$

  With symmetry $t_i = t_k = t^*$. Hence $\alpha_i = 1/n$ for all i and $\tau = t^*$, too. Since $\frac{\partial}{\partial t_i}\sum_k \alpha_k = 0$, the above condition boils down to (4).

- Nash equilibrium: The FOC for country i is given by

$$\frac{\partial T_i}{\partial t_i} = \frac{\partial}{\partial t_i}(\Phi t_i \alpha_i) = \alpha_i t_i \frac{\partial \Phi}{\partial t_i} + \Phi\left(\alpha_i + t_i \frac{\partial \alpha_i}{\partial t_i}\right) = 0.$$

  Using symmetry yields (5).

- ESS: Partially differentiating $T_i - T_j$ with respect to t_i gives:

$$\begin{aligned}\frac{\partial}{\partial t_i}(T_i - T_j) &= \frac{\partial}{\partial t_i}\left(\Phi(t_i\alpha_i - t_j\alpha_j)\right) \\ &= (t_i\alpha_i - t_j\alpha_j)\frac{\partial \Phi}{\partial t_i} + \Phi\left(\alpha_i + t_i\frac{\partial \alpha_i}{\partial t_i} - t_j\frac{\partial \alpha_j}{\partial t_i}\right) = 0.\end{aligned}$$

  In a symmetric situation, $t_i = t_j = t$ as well as $\alpha_i = \alpha_j = 1/n$. Given symmetry of α_j in $\mathbf{t}_{-j}$ gives that $\frac{\partial \alpha_j(t\mathbf{1}^n)}{\partial t_i} = \frac{\partial \alpha_k(t\mathbf{1}^n)}{\partial t_i}$ for all $j, k \neq i$. Symmetry also implies that (2) holds. Using this to replace $\frac{\partial \alpha_j(t\mathbf{1}^n)}{\partial t_i}$ leads to

$$\left.\frac{\partial}{\partial t_i}(T_i - T_j)\right|_{\mathbf{t}=t\mathbf{1}^n} = \Phi\left(\alpha_i + t^E\frac{\partial \alpha_i}{\partial t_i}\left(1 + \frac{1}{n-1}\right)\right) \tag{A.1}$$

  in any symmetric situation. At the ESS, (A.1) has to equal zero. With $\Phi > 0$, this requires (6). •

A.2 Proof of Proposition 2

Calculate the fiscal externality generated by country i on country j as

$$\frac{\partial T_j}{\partial t_i} = \frac{\partial}{\partial t_i}(\Phi t_j \alpha_j) = \alpha_j t_j \frac{\partial \Phi}{\partial t_i} + t_j \Phi \frac{\partial \alpha_j}{\partial t_i}.$$

In a symmetric Nash equilibrium, the α_k as well as the t_k are equal across countries. Moreover, (5) holds. Using this to replace $\frac{\partial \Phi}{\partial t_i}$ gives

$$\frac{\partial T_j(t^N \mathbf{1}^n)}{\partial t_i} = -\Phi\left(\frac{1}{n} + t^N\left(\frac{\partial \alpha_i}{\partial t_i} - \frac{\partial \alpha_j}{\partial t_i}\right)\right) = -\Phi\left(\frac{1}{n} + t^N \frac{n}{n-1}\frac{\partial \alpha_i}{\partial t_i}\right),$$

where we used (2). Since $\Phi > 0$ by assumption, the claim follows by rearranging. •

A.3 Proof of Proposition 4

We only outline the case of positive externalities: $\frac{\partial T_j}{\partial t_i} > 0$ for all $\mathbf{t}$; the case of negative externalities follows *mutatis mutandis*. In a symmetric Nash equilibrium with positive externalities, (8) holds. Now use (A.1) to evaluate that at a symmetric Nash equilibrium

$$\frac{\partial}{\partial t_i}(T_i - T_j)\bigg|_{\mathbf{t}=t^N \mathbf{1}^n} = \Phi \frac{1}{n-1}\left(1 - \frac{1}{n} - \varepsilon_\alpha(t^N)\right) < 0.$$

Hence, given uniqueness of the ESS, it must hold that $t^E < t^N$. Combining this with Proposition 2 proves the claim. •

A.4 Comparative statics for Section 4

Approach. Denote by

$$J_{\mathbf{k}} := \left(\frac{\partial^2 \tilde{\Pi}^n}{\partial k_i \partial k_j}\right)_{i,j=1,\dots,n} \quad \text{and} \quad J_{\mathbf{t}} := \left(\frac{\partial^2 \tilde{\Pi}^n}{\partial k_i \partial t_j}\right)_{i,j=1,\dots,n}$$

the square matrices of the derivatives of (10) with respect to capital investments and tax rates. Then the MNE's tax responses can be calculated as:

$$\mathrm{d}\mathbf{k} = -J_{\mathbf{k}}^{-1} \cdot J_{\mathbf{t}} \mathrm{d}\mathbf{t}. \tag{A.2}$$

We focus on symmetric situations: $t_i = t$ for all i. Then $\tau = t$ and $\alpha_i = 1/n$. From (10) and the symmetry of $\tilde{\Phi}$ and $\tilde{\Pi}^g$, investments are the same everywhere: $k_i = k_j$. Moreover,

$$\begin{aligned} \left.\frac{\partial \tau}{\partial k_i}\right|_{\mathbf{t}=t\mathbf{1}^n} &= \sum_k t_k \cdot \frac{\partial \tilde{\alpha}_k}{\partial k_i} = t \sum_k \frac{\partial \tilde{\alpha}_k}{\partial k_i} = 0, \\ \left.\frac{\partial^2 \tau}{\partial k_i \partial k_j}\right|_{\mathbf{t}=t\mathbf{1}^n} &= t \frac{\partial}{\partial k_j} \left(\sum_k \frac{\partial \tilde{\alpha}_k}{\partial k_i} \right) = 0. \end{aligned}$$

Matrices $J_{\mathbf{k}}$ and $J_{\mathbf{t}}$ are symmetric, implying from (A.2) that

$$\frac{\partial k_i(t\mathbf{1}^n)}{\partial t_i} = \frac{\partial k_j(t\mathbf{1}^n)}{\partial t_j} \quad \text{and} \quad \frac{\partial k_i(t\mathbf{1}^n)}{\partial t_j} = \frac{\partial k_j(t\mathbf{1}^n)}{\partial t_i} \quad \text{for all}\, i \neq j.$$

Specifically,

$$\begin{aligned} \frac{\partial^2 \tilde{\Pi}^n}{\partial k_i^2} &= \frac{\partial^2 \tilde{\Pi}^g}{\partial k_i^2} - \tau \frac{\partial^2 \tilde{\Phi}}{\partial k_i^2} - 2 \frac{\partial \tilde{\Phi}}{\partial k_i} \frac{\partial \tau}{\partial k_i} - \tilde{\Phi} \frac{\partial^2 \tau}{\partial k_i^2} \\ &\overset{\text{(s)}}{=} \frac{\partial^2 \tilde{\Pi}^g}{\partial k_i^2} - t \frac{\partial^2 \tilde{\Phi}}{\partial k_i^2}; \end{aligned} \tag{A.3}$$

$$\frac{\partial^2 \tilde{\Pi}^n}{\partial k_i \partial k_j} \overset{\text{(s)}}{=} \frac{\partial^2 \tilde{\Pi}^g}{\partial k_i \partial k_j} - t \frac{\partial^2 \tilde{\Phi}}{\partial k_i \partial k_j}; \tag{A.4}$$

$$\frac{\partial^2 \tilde{\Pi}^n}{\partial k_i \partial t_i} = -\frac{\partial \tilde{\Phi}}{\partial k_i} \frac{\partial \tau}{\partial t_i} - \tilde{\Phi} \frac{\partial^2 \tau}{\partial k_i \partial t_i} \overset{\text{(s)}}{=} -\frac{1}{n} \frac{\partial \tilde{\Phi}}{\partial k_i} - \tilde{\Phi} \frac{\partial \alpha_i}{\partial k_i}; \tag{A.5}$$

$$\frac{\partial^2 \tilde{\Pi}^n}{\partial k_i \partial t_j} \overset{\text{(s)}}{=} -\frac{1}{n} \frac{\partial \tilde{\Phi}}{\partial k_i} - \tilde{\Phi} \frac{\partial \alpha_j}{\partial k_i}. \tag{A.6}$$

Here, $\overset{\text{(s)}}{=}$ indicates that the expression is evaluated at a symmetric situation ($t_i = t$ for all i). Moreover, the values of expressions (A.3) through (A.6) do not vary across i and

j. Hence, $J_{\mathbf{k}}$ and $J_{\mathbf{t}}$ are of form

$$J_{\mathbf{k}} \overset{\text{(s)}}{=} \begin{pmatrix} a & b & \dots & b \\ b & a & \dots & b \\ \vdots & \vdots & \ddots & \vdots \\ b & b & \dots & a \end{pmatrix} \quad \text{and} \quad J_{\mathbf{t}} \overset{\text{(s)}}{=} \begin{pmatrix} c & d & \dots & d \\ d & c & \dots & d \\ \vdots & \vdots & \ddots & \vdots \\ d & d & \dots & c \end{pmatrix}$$

where $a := \frac{\partial^2 \Pi^n}{\partial k_i^2}$, $b := \frac{\partial^2 \Pi^n}{\partial k_i \partial k_j}$, $c := \frac{\partial^2 \Pi^n}{\partial k_i \partial t_i}$, and $d := \frac{\partial^2 \Pi^n}{\partial k_i \partial t_j}$. The inverse of $J_{\mathbf{k}}$ is:

$$J_{\mathbf{k}}^{-1} \overset{\text{(s)}}{=} \frac{1}{D} \begin{pmatrix} a+(n-2)b & -b & \dots & -b \\ -b & a+(n-2)b & \dots & -b \\ \vdots & \vdots & \ddots & \vdots \\ -b & -b & \dots & a+(n-2)b \end{pmatrix}$$

where

$$D := a^2 + (n-2)ab - (n-1)b^2. \tag{A.7}$$

Solution. Hence,

$$\frac{\partial k_i}{\partial t_i} \overset{\text{(s)}}{=} -\frac{1}{D}\left(ac + (n-2)bc - (n-1)bd\right), \tag{A.8}$$

$$\frac{\partial k_j}{\partial t_i} \overset{\text{(s)}}{=} -\frac{1}{D}\left(ad - bc\right), \tag{A.9}$$

where a to d need to be replaced by the expressions in (A.3) to (A.6).

Composite effects. From this we obtain:

$$\begin{aligned}
\frac{\partial(k_i - k_j)}{\partial t_i} &\overset{(s)}{=} -\frac{1}{D}\left(a+(n-1)b\right)(c-d) = -\frac{c-d}{a-b} \\
&= -\left(\frac{\partial^2\tilde{\Pi}^n}{\partial k_i^2} - \frac{\partial^2\tilde{\Pi}^n}{\partial k_i \partial k_j}\right)^{-1}\left(-t\Phi\left(\frac{\partial\tilde{\alpha}_i}{\partial k_i} - \frac{\partial\tilde{\alpha}_j}{\partial k_i}\right)\right) \\
&= \tilde{\Phi}\cdot\frac{n}{n-1}\left(\frac{\partial^2\tilde{\Pi}^n}{\partial k_i^2} - \frac{\partial^2\tilde{\Pi}^n}{\partial k_i \partial k_j}\right)^{-1}\frac{\partial\tilde{\alpha}_i}{\partial k_i} < 0. \qquad \text{(A.10)}
\end{aligned}$$

Similarly,

$$\begin{aligned}
\frac{\partial K}{\partial t_i} &\overset{(s)}{=} \frac{\partial k_i)}{\partial t_i} + (n-1)\frac{\partial k_j)}{\partial t_i} \\
&\overset{(s)}{=} -\frac{1}{D}\left((a-b)(c+(n-1)d)\right) = -\frac{c+(n-1)d}{a+(n-1)b} \\
&= -\frac{1}{a+(n-1)b}\left[\left(\frac{1}{n}\frac{\partial\tilde{\Phi}}{\partial k_i} - \tilde{\Phi}\frac{\partial\tilde{\alpha}_i}{\partial k_i}\right) - \frac{n-1}{n}\frac{\partial\tilde{\Phi}}{\partial k_i} - (n-1)\tilde{\Phi}\frac{\partial\tilde{\alpha}_j}{\partial k_i}\right] \\
&= -\frac{1}{a+(n-1)b}\left[-\frac{\partial\tilde{\Phi}}{\partial k_i} - \tilde{\Phi}\frac{\partial}{\partial k_i}\sum_j \tilde{\alpha}_j\right] \\
&= \frac{1}{a+(n-1)b}\cdot\frac{\partial\tilde{\Phi}}{\partial k_i} = \left(\frac{\partial^2\tilde{\Pi}^n}{\partial k_i^2} + (n-1)\frac{\partial^2\tilde{\Pi}^n}{\partial k_i \partial k_j}\right)^{-1}\cdot\frac{\partial\tilde{\Phi}}{\partial k_i}. \qquad \text{(A.11)}
\end{aligned}$$

The sign of (A.11) depends on the sign of $\frac{\partial\tilde{\Phi}}{\partial k_i}$, i.e., on whether taxable profits increase or decrease with investment, k_i. Observe from (10) that, in a symmetric situation,

$$\left.\frac{\partial\tilde{\Phi}}{\partial k_i}\right|_{t_i=t} \geq 0 \quad \Longleftrightarrow \quad \frac{\partial\tilde{\Phi}}{\partial k_i} \leq \frac{\partial\tilde{\Pi}^g}{\partial k_i}.$$

Hence, if taxable profits are marginally smaller [larger] than gross, economic profits, then an increase [a decrease] in any tax rate will lead to a reduction of total investment from the economic area.

Separability (used in Section 4.4). For the special case that profits are separable across countries (i.e., $\tilde{\Pi}^n(\mathbf{k}) = \sum_j \left(\pi^g(k_j) - \tau\phi(k_j)\right)$, we get that $b = 0$. Then (A.10) and

(A.11) simplify to

$$\frac{\partial(k_i - k_j)}{\partial t_i} = \phi \cdot \frac{n}{n-1}\left(\frac{\partial^2 \pi^n}{\partial k_i^2}\right)^{-1} \cdot \frac{\partial \tilde{\alpha}_i}{\partial k_i}; \tag{A.12}$$

$$\frac{\partial K}{\partial t_i} = \left(\frac{\partial^2 \pi^n}{\partial k_i^2}\right)^{-1} \cdot \frac{\partial \phi}{\partial k_i}. \tag{A.13}$$

A.5 Proof of Proposition 5

Plugging (12) into (6), we find that an interior ESS t^E for FA and CCCTB is characterized by

$$1 = -t^E \cdot \frac{n^2}{n-1} \cdot \frac{\partial \tilde{\alpha}_i(k \cdot \mathbf{1}^n)}{\partial k_i} \cdot \frac{\partial}{\partial t_i}(k_i - k_j). \tag{A.14}$$

Using (A.10) leads to (14). •

A.6 Proof of Proposition 6

The profit-maximization condition comes from (10), taking into account separability and the fact that, in a symmetric situation, $\partial\tau/\partial k_i = 0$.

Observe that with separability, $\phi = \tilde{\Phi}/n$, $\partial\phi/\partial k = \partial\tilde{\Phi}/\partial k_i$, and $D = \pi''(k) - t\phi''(k)$. The general expressions used in Proposition 1 and its proof can then be simplified, using Eqs. (12) and (13) as well as (A.10) and (A.11). Moreover, (15) holds for weighing formulas. Re-arranging terms gives the results, as claimed.[21] •

A.7 Proof of Proposition 7

Verify that

$$\frac{\partial U_i}{\partial t_i} = \frac{1}{n}\frac{\partial \Pi^n}{\partial t_i}u_c(c_i, g_i) + \frac{\partial T_i}{\partial t_i}u_g(c_i, g_i) \quad \text{and} \quad \frac{\partial U_j}{\partial t_i} = \frac{1}{n}\frac{\partial \Pi^n}{\partial t_i}u_c(c_j, g_j) + \frac{\partial T_j}{\partial t_i}u_g(c_j, g_j).$$

[21] Observe that the ESS condition can directly be adopted from (16).

At a symmetric situation ($t_i = t$ for all i), we get $c_i = c$, $g_i = g$, and $\frac{\partial \Pi^n}{\partial t_i} = \frac{\partial \Pi^n}{\partial t_j}$ for all i, j.

Consider now a symmetric Nash equilibrium, $\mathbf{t}^N = t^N \mathbf{1}^n$. Here, $\frac{\partial U_i(\mathbf{t}^N)}{\partial t_i} = 0$ holds for any i. Consequently, the cross-country spillovers are given by

$$\begin{aligned}\frac{\partial U_j(\mathbf{t}^N)}{\partial t_i} &= \frac{1}{n}\frac{\partial \Pi^n(\mathbf{t}^N)}{\partial t_i} u_c(c^N, g^N) + \frac{\partial T_j(\mathbf{t}^N)}{\partial t_i} u_g(c^N, g^N) \\ &= -u_g(c^N, g^N)\left(\frac{\partial T_i(\mathbf{t}^N)}{\partial t_i} - \frac{\partial T_j(\mathbf{t}^N)}{\partial t_i}\right).\end{aligned}$$

Since $u_g > 0$, a positive [negative] welfare externality prevails if the tax revenue differential between i and j marginally decreases [increases] with the tax rate t_i.

Assuming uniqueness, we therefore obtain

$$t^N \leq t^* \iff \frac{\partial T_i(\mathbf{t}^N)}{\partial t_i} - \frac{\partial T_j(\mathbf{t}^N)}{\partial t_i} \leq 0. \tag{A.15}$$

This proves the second item of the claim. By its symmetry, an interior ESS is characterized by

$$\frac{\partial (U_i(\mathbf{t}^E) - U_j(\mathbf{t}^E))}{\partial t_i} = u_g(c^E, g^E)\left(\frac{\partial T_i(\mathbf{t}^E)}{\partial t_i} - \frac{\partial T_j(\mathbf{t}^E)}{\partial t_i}\right) = 0.$$

Hence, from (A.15),

$$\begin{aligned} t^N < t^* &\iff \frac{\partial T_i(\mathbf{t}^N)}{\partial t_i} - \frac{\partial T_j(\mathbf{t}^N)}{\partial t_i} < 0 = \frac{\partial T_i(\mathbf{t}^E)}{\partial t_i} - \frac{\partial T_j(\mathbf{t}^E)}{\partial t_i} \\ &\iff t^E < t^N. \end{aligned}$$

The converse holds if $t^N > t^*$. This settles the proof. •