Abstract: Regulations concerning old-age (social security) provisions for different subgroups of the population vary considerably throughout the world, ranging from uniform schemes for the whole population to a largely diversified palette for different population subgroups. While pension architectures are typically driven by forces of political economy, this paper investigates into efficiency-based rationales for and against uniform or group-specific pension schemes. A series of small OLG models shows that no clear-cut answer exists as to the superiority of either arrangement. Key factors for any such choice include the mobility of individuals between groups, the additivity properties of the pension game, and the decision rules for pension design.

JEL-classification: H55.

Keywords: Social security, Pay-as-you-go Schemes, Heterogeneity.
1 Introduction

Should there be a uniform old-age pension scheme for the whole population? Or should society operate different social security schemes for different subgroups in the population? And if so, how should such a separation proceed?

A superficial overview on today’s pay-as-you-go (PAYG) pension arrangements around the world suggests — at least if one interprets observed pension structures as revealed social preferences — that there are many and quite different ways how to answer these questions. Regulations concerning the membership structure, the coverage, and the range of schemes differ largely across countries, ranging from universal or uniform systems which cover the whole population to group-specific or selective\(^1\) ones which are not only restricted to the labor force but, within that, possibly provide different pension arrangements for different occupational and professional groups. Most prominent lines of separation for pension schemes run between civil servants, farmers, blue- and white-collar workers, and self-employed, but some pension architectures provide more than a hundred subdivisions (see Section 2 for more details).\(^2\)

This diversity in pension arrangements triggers questions both with respect to its causes and to its effects (in particular, with respect to “optimality”): Why do and did countries adopt different pension architectures? And when is it preferable to operate a uniform scheme rather than a selection of group-specific ones?

Starting with the causes, a most prominent answer seems to be that pension schemes have emerged “historically”. Apart from almost being tautological, this answer is unsatisfactory since, drawn with a broad brush, most countries followed roughly the same historical routes when constructing and amending their pension architectures: They typically started the establishment of old-age provisions for civil servants and employees in public enterprises, followed by employees in large firms or certain key industries (miners, steelworkers etc.), gradually extended eligibility to employees in smaller firms, less important branches, and the service sector, and in some cases also to farmers, the self-employed

\(^1\)Terminology is neither well-defined nor standardized here. “Selectivity” need not be understood in the sense of group-specific but can also refer to certain conditions of eligibility for social benefits (e.g., having to pass a means test before drawing on pensions). As well, universality need not refer to universal coverage but may, in the sense of Beveridgean schemes, refer to benefits that are invariant across beneficiaries.

\(^2\)Lines of separation other than “corporatist” ones are rare. One might, e.g., use other and more natural distinctive criteria like gender – which in many countries would be prohibited by equality principles – or age as, e.g., discussed in Diamond (2003).
and the not-working part of the population. However, while some countries chose to set up different (and sometimes quite numerous) pension schemes for various subgroups, others incorporated new subgroups into existing schemes or, at some point in time, decided to merge several schemes. Why some countries managed to “harmonize” their pension arrangements while others failed or have not even seriously tried it is probably a matter of political economy in (what is seen as) a redistribution game. Notwithstanding the fact that pension design is driven by political forces or distributional concerns, as an economist one might reflect on what would be an “optimal” architecture from an allocative perspective. In this paper, we try to shed some light on that question. Remarkably, there do not exist many models upon which such an analysis could be based; economic literature on universal versus group-specific pension schemes is very scarce (see Section 3 for a brief review).

Basically, our analysis proceeds through a series of examples each of which is tailored to a standard OLG model of (PAYG) social security. Unlike most of the pensions literature, we introduce some heterogeneities among pensioners. The specific cases we consider are the following: (i) a two-sector economy with intersector mobility, very much in the spirit of the pension- or tax-harmonization literature; and (ii) a stochastic OLG-model where individuals have different risk characteristics and/or risk preferences.

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3The French history provides an instructive example of corporatist obstruction of pension reforms: In the late 1940s, the French government aimed at creating, as an expression of “national solidarity”, a large and uniform social security scheme, called régime général. However, a number of groups fiercely opposed that proposal with the effect that a unified scheme covering the entire population was never reached:

- Self-employed, craftsmen and farmers, at that time a substantial part of the working population and a strong political force, wanted to keep their independent status and refused to be assimilated with wage-earners. As a result of their protest, they were allowed to set up separate pension schemes for farmers, craftsmen and self-employed.

- Some groups such as civil servants, miners, railwaymen, etc already had their own schemes which provided more generous benefits than the proposed régime général. As a result of their protest, these groups’ régimes spéciaux were allowed to remain in existence “temporarily” — an interim solution that by and large has survived until today.

- The “cadres” (executives) did not want to be included in an all-embracing scheme either. The result was a compromise: They had to enter the régime général with contributions on their wages up to the social security ceiling, while for incomes above that ceiling they established a mandatory scheme on their own.

(L’Observatoire des Retraites, 2003).
Whereas in the first example it turns out that a uniform or harmonized (across population subgroups) scheme is unambiguously preferable the implications of the other example is mixed: In the second example efficiency is compatible with both uniform or group-specific pension schemes. In the third example, anything goes, depending on the scenario. Hence, conclusive answers concerning the preferability of a uniform or a group-specific pension architecture are not feasible. While the lack of institutional richness in our stylized examples does not allow for specific answers on certain policy issues, our analysis suggests that it is difficult to base proposals to “harmonize pensions” – be it across countries or across population subgroups – on economic (i.e., efficiency-based) arguments.

The rest of this paper is organized as follows: Section 2 portrays some of the institutional richnesses that can be found in pension arrangements across and within countries. Section 3 reviews the related literature on our theme. Section 4 then collects three specific set-ups in which the question of uniform vs. group-specific pensions schemes can be discussed in a (hopefully) meaningful way. Section 5 concludes.

2 Some institutional details

2.1 General Remarks

Social security systems can broadly be classified into two groups (Pieters, 1997): universal and selective ones. Universal systems, which are (cum grano salis) practised in the Netherlands, Sweden, Denmark, or Japan cover the whole population, while selective ones only cover the working population or parts thereof. Within the latter group again two types can be distinguished: (a) basic income systems that uniformly cover all employed persons as in the United Kingdom or Ireland, and (b) systems with specific and separate schemes for different groups of the work force (in- or excluding entrepreneurs or the self-employed). Examples for (b) can be found in many European countries such as Germany, Austria, France, Italy, Belgium, Greece, Spain or Portugal. Some countries such as Japan combine a universal “basic” pension with group-specific mandatory pension arrangements for entrepreneurs, civil servants, and employees (National Institute, 2004). This is the broad picture. Within almost each of the national systems the rules concerning participation (compulsory vs. mandatory) and contribution (fixed or voluntary) typically discriminate against different occupational and professional groups.

Across countries that operate group-specific compulsory public pension schemes, the
number of schemes varies widely, ranging from four in Germany (civil servants, farmers, miners and employees) and six in Austria (before 2005) over more than 50 in Italy (before the Dini reform in 1995/96) and Greece to over a hundred in France (out of which “only” 26 are open while the others are heading for extinction). These schemes are distinguished from one another by special regulations, financing, possibly contribution rates and methods of pension calculation and adjustment. To get an idea of the possible bandwidth, let us briefly have a more detailed look at the German, Austrian and French pension architectures.

2.2 Germany

The biggest block in the statutory pillar of the German pension architecture (the Gesetzliche Rentenversicherung – GRV) is a PAYG scheme and generally designed for employed wage and salary earners. While “ordinary” state employees (for whom standard labor legislation applies) are included, civil servants, soldiers and judges are not; their pensions are tax- rather than contribution financed and meant to also function as occupational pensions. The latter also holds for the miners’ pension scheme (Knappschaftsversicherung) which levies higher contributions than the main GRV scheme. Entrepreneurs and the self-employed are typically not compulsory members of the GRV. However, there is a remarkable list of special arrangements:

- Lawyers, doctors, dentists, veterinary surgeons, pharmacists, auditors, architects, and some other professional groups of self-employed have separate compulsory pension schemes (called Versorgungswerke) outside the GRV. These schemes are funded, but contributions and benefit rules very closely follow those of the GRV.\(^4\)

- Self-employed individuals in agriculture, forestry and mainland fishery as well as their spouses are compulsory members of a separate pension scheme for farmers (Alterssicherung der Landwirte). Contributions are lump-sum (in 2003: 198 Euros/month), and pensions are proportional to the duration of membership in the scheme, independent of previous incomes, but contingent on retirement from the farm. The scheme is highly tax-subsidized (see Deisler, 1998, for details).

- Self-employed craftsmen, who have to be enrolled in a guild, are compulsory members of the GRV (with the possibility to opt out after 18 years of membership).

\(^4\)For details see Jung (1998). Currently, there exist about 80 separate pension schemes for the various professional groups with a total of 560,000 members.
They pay the full contribution (employers’ plus employees’ share), levied either on an average earnings base or on actual taxable income. Similar compulsion and regulations apply to some other professional groups such as teachers, midwives, nurses, pilots, in-shore fishermen, artists and journalists (if self-employed).

- Artists, writers and journalists are, when self-employed, compulsory members of the GRV. They only pay the employees’ share of the contribution. The rest is financed out of the federal budget (20% since 2000) and by the Künstlersozialabgabe (80%), a tax which is levied (currently at a rate of 4%) on the expenses for artists’ fees and royalties of those who market the products of artists and writers (e.g., publishers, theatres, orchestras, broadcasting firms, galleries). The scheme is notorious for high administrative costs and low compliance (for details see Zacher and Zacher, 2000).

- The remaining self-employed are not obliged to join any pension scheme; old-age provisions are at their own responsibility. They have, however, options to join the GRV either as a regular or as a voluntary member. The “regular”-option implies – with some ramifications – income-dependent contributions at the standard conditions; joining voluntarily leaves one with free choice of one’s (lump-sum) contribution.5

Germany, thus, hosts a fancy assortment of specific old-age provisions for different population subgroups (arranged along professional lines). They range from compulsory membership in a PAYG scheme to full freedom over old-age provisions.

2.3 Austria

In Austria, almost 96% of the working population are covered by mandatory and publicly administered social security schemes. In November 2004, Austria undertook a pension reform. Prior to the reform, the pension architecture was subdivided into six different schemes (see Holzmann and Heitzmann, 2002):

(i) for workers and employees (with special regulations for miners);

(ii) for self-employed and entrepreneurs, including journalists, artists, dentists, veterinary doctors and some others;

5Voluntary members of the GRV are treated differently from regular members with respect to disability pensions and early retirement.
(iii) for medical doctors, pharmacists and patent lawyers;
(iv) for farmers, forestry farmers, and their family members;
(v) for notaries; and
(vi) for civil servants.

These schemes differ(ed) from one another with respect to contribution and (implicit) replacement rates, methods of pension calculation and adjustment, and eligibility and early retirement rules.

Under the label of “harmonization”, the 2004 Austrian pension reform set out to unify these six different schemes. After some (quite long) transition period, the full working population should be subject to identical pension regulations, the core of which is summarized as “45-65-80”, meaning that if one retires, after 45 years of work, at the age of 65 one should draw 80% of one’s average lifetime salary as a pension. In its present form, the “harmonization” is incomplete, leaving to farmers, civil servants and other typical “spongers” at least parts of their privileges they held before the reform.

2.4 France

The French system of old-age provisions has a highly complex architecture with numerous subdivisions, by and large organized along occupational and professional groups, but also for certain larger companies (for a survey see, e.g., ADECRI, 2003).

In principle, the first tier of the pension architecture comprises both means-tested flat rate pensions (minimum vieillesse) of marginal importance, complemented by mandatory earnings-related pensions. The most important of scheme is the so-called régime général. In addition, there are 122 separate other régimes, each independently organised with its own (PAYG) budget and its own regulations for benefits and contributions (typically, they are defined-benefit schemes). There are separate schemes for private employees

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6 The second tier is made up of complementary pensions (such as ARGIC or ARCCO). They are legally mandatory, pay-as-you-go financed, of the defined-contribution type, organized on a nationwide basis, and provide on average around 40% of old-age incomes.

7 There are, however, financial transfers between the schemes in order to neutralize demographic or sociological disparities that unfavorably hit single régimes. However, these compensation schemes are intransparent and messy and basically hampered by the attempt to avoid financing by others of particular “advantages” that the régimes may have established for their clienteles.
(including one for farm workers) with a distinction between “normal” employees and such with management tasks (so-called cadres), entrepreneurs, trades people, twelve different schemes for various groups of self-employed professionals, civil servants, farmers, miners, and workers employed by public sector corporations such as SNCF (national railways), EDF-GDF (electricity and gas) or RATP (Parisian public transport). The social security offices in France have an autonomous status; they do not merge with the state but are managed, under state supervision, by employers’ associations and trade unions.

3 Related Literature

In spite of the colorful varieties of reality, most of the theoretical literature on social security only considers set-ups with homogeneous populations and one-sector economies — where diversity issues naturally cannot arise. But even in papers that allow for heterogeneity across agents, the typical assumption is that everybody in the economy is subject to the same sort of pension regulations and old-age provisions and that the economy operates under a single PAYG budget constraint (see, e.g., Groezen et al., 2002; Conesa and Krueger, 1999). We may note some exceptions, however:

- There exists a literature on social security schemes in the presence of factor mobility (e.g., Homburg and Richter, 1993; Razin and Sadka, 1999; Breyer and Kolmar, 2002; Perotti, 2001). It deals with the question whether distinct national pension schemes can and should survive or rather being harmonized. In the set-ups discussed in that literature, “harmonization”, by and large, corresponds to a uniform scheme for the integrated economy while keeping distinct national schemes would amount to group-specificity according to location or residence. The main observation of that literature (with Breyer and Kolmar (2002) and Perotti (2001) as exceptions) is that efficiency can only be obtained in the absence of distortionary wedges — which calls for a harmonized or uniform pension system.

- Wagener (2000) considers a social-security model with two types of individuals: entrepreneurs and workers (see Steinberger (2005) for a similar analysis). Entrepreneurs hire workers, pay them a deterministic wage while earning stochastic net profits themselves. The income of an entrepreneur is, thus, more risky than that of a wage earner. Individuals endogenously choose whether to become a worker or an entrepreneur and, in the equilibrium, the numbers of individuals in each category and
the non-stochastic wage rate will adjust such as to balance demand and supply of workers. In such a setting, a PAYG pension schemes can be understood as a tool to risklessly transfer resources across time. One may now ask whether such an old-age income insurance scheme would be welcome by workers (with “safe” careers) or entrepreneurs (with risky income streams). First (and not too surprisingly), Wagener (2000) shows that attitudes differ across workers and entrepreneurs — which indicates that a one-size-fits-all pension scheme can, at least in certain circumstances, be Pareto-improved upon by group-specific schemes. Second (and maybe a bit more surprisingly), it is unclear which of the two groups has a higher willingness-to-pay for old-age income insurance, implying that the widespread practice to not include entrepreneurs and the self-employed might indeed rest on false grounds.

- The significance of uniformity versus selectivity in pension schemes for efficiency reasons also emerges from the debate on a Pareto-improving transition from PAYG to funded schemes. As argued by Brunner (1994) for an open and by Brunner (1996) for a closed economy, if households differ in their earnings abilities but are subject to a uniform PAYG pension scheme that distorts their labor-consumption choices, then a Pareto-improving transition from PAYG to funding may be impossible – while such a transition would be feasible with a homogeneous population, as demonstrated by Homburg (1990) for an open economy or by Breyer and Straub (1993) for a closed one. The reason for that impossibility is that, in order to maintain the utility levels of the initial steady state during transition, one might have to use type-specific tax-transfer rates and, thus, leave the uniformity set-up. In addition, implementing type-specific transition policies may be informationally infeasible.

4 Two Small Models

4.1 Notation

In the sequel, we consider intertemporal economies with two active overlapping generations. Periods are indexed by \( t \in \mathbb{N}_0 \). Each single individual lives for two active periods, called “working age” and “old-age”. We will refer to generation \( t \) as all individuals who spend their working age in period \( t \). We will denote the cohort size of generation \( t \) by \( N_t := |N_t| \).
Despite from being born at different dates, individuals may be of different types even within a generation. One might think of a person’s type as the professional group to which he belongs, his productivity, an income class or some other characteristic that may be of relevance in pension arrangements and along which group-specific pension schemes can be arranged. We will index types by \( j \); two types will be enough for our purposes: \( j \in \{1, 2\} \). We denote the number of individuals in generation \( t \) that has type \( j \) by \( N_t^j \); naturally, \( N_t^1 + N_t^2 = N_t \).

Demographic structures and processes are assumed to be very simple. Individuals within a population subgroup do not differ in their reproductive behaviour. More specifically, we assume that each single member of cohort \( t \) and type \( j \) gives birth to one child, resulting in a population growth rate of zero: \( N_t^j = N_t^j \).

Individuals are interested in consumption during the two periods of their lives (no altruism, not bequests). Denote by \( c_t^j \) and \( d_t^j \) the consumption levels of a member of generation \( t \) with characteristics \( j \) during working age (\( c \)) and during old-age (\( d \)). Preferences over consumption of an individual with characteristic \( j \) will be given by

\[
u^j = u^j(c_t^j, d_t^j).
\]

Utility functions are assumed to be well-behaved and endowed with the usual monotonicity, concavity and limit properties.

By a pension scheme we mean a PAYG-type intergenerational transfer scheme with a binding periodwise budget constraint. Generally, a pension scheme is characterized by two characteristics: its members and a set of rules that determines the contributions which individuals have to make during their working age and the pension they can draw during their old age.

As our examples only consider two different types of individuals, we will also only consider two types of pension architectures: one with group-specific pension schemes — where membership is open only to one of the two types — and one with a uniform scheme that applies the same set of rules to both types of individuals. Note that within each scheme, every member is treated identically. In our context, the “set of rules” will simply boil down to a one-dimensional parameter, namely the level of pension contributions. Once this has been set, pension levels follow endogenously via demographics and the balanced-budget requirement.
4.2 Example 1: Pensions with Mobility

Consider a standard 2-OLG economy with identical individuals who can choose between two types \((j = 1, 2)\) of jobs, white-collar work and blue-collar work, say. Denote by \(L_j^t\) the number of individuals in job \(j\). Together with capital and at any period \(t\), labor contributes to the production of national output \(Y\) via a standard and stationary neoclassical and constant-returns-to-scale (CRS) production technology \(F\):

\[ Y_t = F(L_1^t, L_2^t, K_t) \]

Initially, individuals within each cohort are identical. They can choose between taking a blue- or a white-collar job. Labor market equilibrium, thus, requires

\[ L_1^t + L_2^t = N. \tag{1} \]

Workers earn gross wages \(w_j^t\) (for \(j = 1, 2\)) which, if the economy is organized through competitive markets, are determined by marginal productivities:

\[ w_j^t = F_{L_j}(L_1^t, L_2^t, K_t) \]

for \(j = 1, 2\). The economy can run a universal PAYG-scheme or two differentiated ones for white- and blue-collar workers. If in period \(t\) the pension scheme levies a per-capita contribution of \(b_j^t\) and pays out a per-capita pension \(p_j^t\), then these are related by the budget constraint

\[ b_j^t = p_j^t \]

(recall that there is no population growth). A uniform pension scheme would be characterized by \(b_1^t = b_2^t\) for all \(t\).

The intertemporal consumption choice for an individual reads as

\[
\max_{c_j^t, d_j^t} u(c_j^t, d_j^t) \tag{2}
\]

subject to

\[ c_j^t + \frac{d_j^t}{1 + r_{t+1}} \leq w_j^t + b_j^t \cdot \left( \frac{1}{1 + r_{t+1}} - 1 \right). \]

The fraction in bracket term is the Aaron-factor \(\alpha\) of the pension scheme to which the individual is a member. Solving (2) yields a value-function \(v_j^t = v(w_j^t + b_j^t(\alpha - 1))\) where the present-value of lifetime income enters in a strictly monotonically increasing way.
When households are free to choose jobs, an equilibrium requires the marginal individual to be indifferent between the two options. I.e.,

\[ w_t^1 + b_t^1(\alpha - 1) = w_t^2 + b_t^2(\alpha - 1). \]  

(3)
must hold. On the other side, intertemporal Pareto-efficiency requires production to be organized at any point in time such as to have marginal productivities of the two types of labor equalized:

\[ F_{L1}(L_t^1, L_t^2, K_t) = F_{L2}(L_t^1, L_t^2, K_t) \]

for all \( t \). Obviously, this is only compatible with (3) if

\[ b_t^1 = b_t^2 \]

for all \( t \), i.e., if the two pension schemes are harmonized — or if there is only one uniform scheme for all. In the absence of any administrative costs (as assumed), harmonizing pension schemes or running a uniform scheme are allocationally equivalent. Either option is superior to having group-specific pension schemes. If one would assume that operating a pension scheme involves some cost subadditivity with respect to the number of its participants, then a uniform pension scheme would be the best among all feasible pension arrangements.\(^8\)

The simple model just presented also applies, with appropriate modifications, to an international or a multisector framework with free labor mobility between countries or sectors. Let us illustrate this for the international case; the multi-sector case can be constructed simply by re-labelling “countries” to “sectors”. The analysis basically boils down to the “harmonization issue” discussed in Homburg and Richter (1993):

In a perfectly integrated economy, there are two countries 1 and 2 with standard CRS production functions \( F^j(L_t^j, K_t^j) \). Here, \( L_t^j \) and \( K_t^j \) denote, respectively, labor and capital input in country \( j \) in period \( t \). Each country can run a PAYG-scheme where contributions \( b_t^j \) and pensions \( p_t^j \) are related by the nationwise budget constraints

\[ b_t^j = p_t^j \cdot (1 + n_t^j). \]

\(^8\)Note, however, that this does not imply that having a PAYG-scheme at all is welfare-improving. For the current framework, e.g., it is well-known that whenever the Aaron-factor \( \alpha \) is less than unity (dynamic efficiency), the optimal value of \( b \) (and of any \( b_t^j \) in group specific schemes) would be zero.

11
By \( n_j^t := L_j^t / L_{j-1}^t \) we denote the period-\( t \) growth rate of the labor force in country \( j \) (which may differ across countries). Individuals are free to migrate before entering the labor market. A labor market equilibrium in \( t \) requires (1) and

\[
 w_1^t + b_1^t (\alpha_1^t - 1) = w_2^t + b_2^t (\alpha_2^t - 1)
\]  

(4) to hold. Again, \( \alpha_j^t = \frac{1 + n_j^{t+1}}{1 + r_j^{t+1}} \) is the Aaron-factor of scheme \( j \). Profit maximization by firms implies \( L_j^t (L_j^t, K_j^t) = w_j^t \). Efficiency requires (among other conditions) that marginal productivities of labor are equalized across countries: \( F_1^t (L_1^t, K_1^t) = F_2^t (L_2^t, K_2^t) \) for all \( t \).

It is obvious then that, starting from a situation where \( n_1^t = n_2^t = n \), harmonization of national schemes \( b_1^t = b_2^t \) leads to efficiency. Moreover, as argued by Razin and Sadka (1999), if the schemes are harmonized, then the hypothesis of equal population growth rates \( n_1^t = n_2^t = n = 0 \) is compatible with a perfect-foresight equilibrium of the economy (which in turn then is production-efficient). As before, the harmonization of schemes can alternatively be interpreted as running a uniform scheme for the two countries.

In this example, the case for uniform rather than group-specific schemes is based on the avoidance of distortions. The effect of the pension scheme is like that of a tax or a subsidy. Similar as in standard models of taxation, if group-specific (i.e., non-harmonized) schemes drive a wedge between things that should, in an efficient allocation, be equalized (such as marginal productivities or marginal rates of substitution), then it ought to be avoided.

### 4.3 Example 2: Old-age provisions as portfolio problems

#### 4.3.1 Stochastic Set-up

It is well-known (see, e.g., Gale, 1991; Merton, 1983; Hauenschild, 1999; Wagener, 2003) that there might be a rationale for an economy to run a PAYG pension scheme for reasons of intergenerational risk-sharing and diversification.

Consider the following small open 2-OLG economy under uncertainty which is inspired by Thøgersen (1998) and Wagener (2003): At any point in time, there are two groups of equal size (each normalized to unity) in the society. Everybody has an exogenous wage income \( w \) in the first period of his life which he can consume immediately, save for old-age consumption, or contribute to a (compulsory) PAYG pension scheme. The rate of return on saving, which is assumed to be the same across individuals, is risky. We also assume
that the rate of return on the pension scheme is stochastic, but that the rates of return possibly differ across the two groups. Think, e.g., of groups 1 and 2 as different professional groups or employees in different branches. Performance of the pension scheme will then depend on the development of wages or incomes in these groups which may be subject to different stochastic trends (e.g., coal miners vs. IT people, farmers vs. industrial workers).

Indicating random variables by tildes, the intertemporal budget constraints for individuals can be written as:

\[
c_i^j = w_t - b_i^j - s_i^j \\
\tilde{d}_i^j = \tilde{R}_{t+1} \cdot s_i^j + \tilde{\pi}_{i+1}^j \cdot b_i^j.
\]

Here, \(\tilde{R} = 1 + \tilde{r}\) indicates the random return on saving while \(\tilde{\pi}^j\) is the return on the PAYG pension scheme for a member of group \(j\).

For simplicity, we assume that preferences can be represented by a linear-quadratic function of consumption during working age, the expected value of consumption during old-age, and the standard deviation of the latter.\(^9\) In particular, we assume that

\[
u_i^j = u^j(c_i^j, (\tilde{d}_i^j)) = c_i^j + \left( E(\tilde{d}_i^j) - \frac{\gamma_j}{2} \cdot \text{Var}(\tilde{d}_i^j) \right) \tag{5}
\]

where \(\gamma_j > 0\) is a risk-aversion parameter for group \(j\), and \(E\) and \(\text{Var}\) indicate the expected value and the variance of a random variable. Observe that there is no rate of time preference, which will be paralleled below by corresponding assumptions on the rates of returns of old-age provisions; nothing would change by assuming a positive rate of time preference.

With this specification of preferences it suffices to specify the first moments and the variance-covariance matrix of the joint distribution of \((\tilde{R}_t, \tilde{\pi}_1^j, \tilde{\pi}_2^j)\). We make the following assumptions:

- The joint distribution of \((\tilde{R}_t, \tilde{\pi}_1^j, \tilde{\pi}_2^j)\) is time invariant and there is no intertemporal correlation.
- \(E(\tilde{R}_t, \tilde{\pi}_1^j, \tilde{\pi}_2^j) = (\mu_R, 1, 1)\) with \(\mu_R > 1\). This conveys, first, that none of the PAYG schemes is “profitable” in expected terms while, second, that saving has a positive expected rate of return. Moreover, the assumption that \(E\pi_1^j = 1\) ensures that the

\(^9\)The \((\mu, \sigma)\)-approach has, although with more general utility functions, been used in pension modelling, e.g., by Thøgersen (1998) or Wagener (2003).
budgets both for group-specific and for the uniform PAYG pension schemes (to be introduced below) will be balanced on expected terms.

- The variance-covariance matrix is given by

\[
\Sigma = \begin{pmatrix}
\sigma_R^2 & \sigma_{R1} & \sigma_{R2} \\
\sigma_{R1} & \sigma_1^2 & \sigma_{12} \\
\sigma_{R2} & \sigma_{12} & \sigma_2^2
\end{pmatrix}
\]

where positive diagonal elements denote the variances of the returns on saving and on the two PAYG-schemes, and the off-diagonal elements denote, with obvious notation, the covariances between these three “assets”.

An old-age portfolio of an individual can be represented by a vector \((s_t, \alpha_1^t, \alpha_2^t)\) where \(\alpha_j^t\) denotes the amount that is “invested” in the PAYG-scheme with return \(\tilde{\pi}_j^t\). The expected return to the old-age portfolio (and, thus, the expected value of old-age consumption) then is

\[
E(\tilde{d}_j^t) = (s_t, \alpha_1^t, \alpha_2^t) \cdot (\mu_R, 1, 1)' = s\mu_R + \alpha_1^t + \alpha_2^t
\]

and its variance is given by

\[
\text{Var}(\tilde{d}_j^t) = (s_t, \alpha_1^t, \alpha_2^t) \cdot \Sigma \cdot (s_t, \alpha_1^t, \alpha_2^t)'.
\]

### 4.3.2 Pension architectures

We again distinguish two pension architectures:

**Group-specific schemes.** There are separate schemes for the two groups: (Compulsory) contributions \(b_j^t\) made by individual \(j\) carry the rate of return \(\tilde{\pi}_j^t\). Given that the expected rate of return is one, it is ensured that the PAYG-budget balances in the sense that with an invariant pension contribution \((b_j^t = b^j\) for all \(t\)) we get

\[
E(\mu_{t+1}^j) = E(\tilde{\pi}_j^t) \cdot b_j^t = b_j^t = b^j.
\]

The old-age portfolio of a member of group 1 then is given by \((s_1^t, b^1, 0)\) and for a member of group 2 by \((s_1^t, 0, b^2)\). The corresponding values for the expected value and the variance of old-age consumption can be calculated by use of (6) and (7). In particular, expected old-age consumption amounts to \(s_1^t\mu_R + b^j\) while the variance obtains as:

\[
\text{Var}(\tilde{d}_j^t) = (s_1^t)^2 \cdot \sigma_R^2 + (b^j)^2 \cdot \sigma_j^2 + 2s_1^t b^j \sigma_{Rj}
\]

for \(j = 1, 2\).
**Uniform scheme.** There is a single contribution $b$ for both groups. All contributions are pooled and carry the corresponding rate of return. As we assume equal group sizes, this amounts to an old-age portfolio of $(s_j^t, b/2, b/2)$ for a member of group $j$ (differences across groups may still arise from different saving behaviour). Again, the assumption of zero excess return to the PAYG scheme ensures budget balance of the scheme on average. Employing (6) and (7), expected old-age consumption amounts to $s_j^t \mu_R + b$ while the variance amounts to:

$$\text{Var}(\tilde{d}_j^t) = (s_j^t)^2 \cdot \sigma_R^2 + \frac{b^2}{4} \cdot (\sigma_1^2 + \sigma_2^2 + 2 \sigma_{12}) + s_j^t \cdot b \cdot (\sigma_{R1} + \sigma_{R2}).$$

for $j = 1, 2$.

**4.3.3 Individual Choices**

Individuals maximize their utility (5) by choice of saving, taking the pension scheme and its implication for consumption stochastics as given. We assume that saving is constrained to be non-negative. Denote by

$$v^j(b) := \max_{s_j^t \geq 0} \left\{ c_j^t + \left( E(\tilde{d}_j^t) - \frac{\gamma_j}{2} \cdot \text{Var}(\tilde{d}_j^t) \right) \right\} | (6), (7)$$

the indirect utility for a member of group $j$ as a function of the contribution rate $b$ to the PAYG scheme.

- **Group-specific schemes.** Let us briefly explain the interaction between saving and the PAYG scheme for the case of group-specific pension schemes. Solving the savings problem, given a PAYG contribution $b^j$, yields a savings function

$$s^j(b^j) = \max \left\{ 0, \frac{\mu_R - 1 - 2\gamma_j b^j \sigma_{Rj}}{2\gamma_j \sigma_R^2} \right\}.$$ 

Thus, the individual saves more the higher the excess return $\mu_R - 1 > 1$ and the lower the riskiness $\sigma_R^2$ of saving. If the returns on saving and of the PAYG system are positively [negatively] correlated (i.e., $\sigma_{Rj} > 0[< 0]$), the existence of a pension scheme will reduce [increase] saving for reasons of diversification.

Plugging savings into the utility function (5) yields indirect utility as a function of $b^j$ (and the distribution parameters). From maximizing that with respect to $b^j$ we can infer the individual’s preferred scheme. Straightforward calculations produce the following result:

$$b^{j*} = \max \left\{ 0, -\sigma_{Rj} \cdot \frac{\mu_R - 1}{2\gamma_j \cdot (\sigma_R^2 \sigma_j^2 - \sigma_{Rj}^2)} \right\}.$$  

(10)
Hence, a PAYG scheme will be welcome \((b^{i*} > 0)\) if and only if the returns to saving and the PAYG scheme for group \(j\) are negatively correlated \((\sigma_{Rj} < 0)\).\(^{10}\) Individuals in group \(j\) would then like to hold the PAYG asset for reasons of portfolio diversification (reduction of consumption variability) even though it has lower expected return than saving.\(^{11}\)

If group \(j\) operates its own PAYG scheme, it will optimally choose to run it under rule (10). Plugging \(b^{i*}\) into (indirect) utility then yields the maximum amount of utility that group \(j\) could obtain with a group-specific pension scheme. Denote this by

\[
V^j_G := \max_{b^j} v^j(b^j).
\]

- **Uniform scheme.** With a uniform pension scheme, the PAYG contribution is uniformly set to \(b\). Then optimal saving of an individual of type \(j\) amounts to

\[
s^j(b) = \max \left\{ 0, \frac{\mu_R - 1 - \gamma^j b (\sigma_{R1} + \sigma_{R2})}{2\gamma^j \sigma^2_{R1}} \right\}
\]

This will differ from saving under a group-specific scheme with equal contribution \((b^j = b)\) if and only if the two schemes differ in their correlation with savings (i.e., if \(\sigma_{R1} \neq \sigma_{R2}\)).

The problematic part now is to determine how the level of the uniform pension scheme is chosen. Groups potentially differ in their preferences (the degrees of risk aversion \(\gamma^j\) need not be identical). In our discussion below, we therefore distinguish two cases: identical preferences and different preferences.

**4.3.4 Identical preferences**

If individuals have identical preferences \((\gamma^1 = \gamma^2 = \gamma)\), there is no problem of finding the optimal uniform pension scheme. Individuals have identical savings levels from which identical indirect utilities as function of \(b\) can be obtained. Maximizing indirect utility

\(^{10}\)Observe that \(\sigma^2_{R1} - \sigma^2_{Rj} = \rho^2 \sigma^2_{Rj}\) is equal in sign to \(1 - \rho^2\) with \(\rho\) as the coefficient of correlation between \(\hat{R}\) and \(\hat{\pi}^j\).

\(^{11}\)Thus, our simple model is indeed capable of providing an economic rationale for running a PAYG scheme (and one where the Aaron-factor in expected terms falls below unity).
with respect to $b$ yields, after some calculations, the following unanimously most-preferred choice of pension scheme:

$$
b^* = \max \left\{ 0, -\left(\sigma_{R1} + \sigma_{R2}\right) \cdot \frac{\mu_R - 1}{\gamma \cdot (\sigma^2_R(\sigma^2_1 + \sigma^2_2 + 2\sigma_{12}) - (\sigma_{R1} + \sigma_{R2})^2)} \right\}. \tag{11}$$

Denote the utility level for individual $j$ that results with (11) by $V^j_U$. Since preferences and incomes are identical, we naturally obtain $V^1_U = V^2_U$.

A uniform pension scheme is Pareto-better than a selection of group-specific schemes if and only if

$$V^j_U \geq V^j_G$$

holds for $j = 1, 2$ with at least one strict inequality. Conversely, if the reverse of (12) holds for $j = 1, 2$, then group-specific schemes will be Pareto-better than a uniform scheme. Finally, if neither (12) nor its reverse hold for both $j$, pension arrangements cannot be ranked according to the Pareto-criterion and, thus, both are Pareto-efficient. As we will argue below, any case might occur.

- A simple case is where the returns $\tilde{\pi}^j$ to the group-specific schemes are identically distributed (i.e., $\sigma_1 = \sigma_2$ and $\sigma_{R1} = \sigma_{R2}$) and less than perfectly positively correlated (i.e., $\sigma_{12} < \sigma_1 \sigma_2$). Then a uniform scheme will always Pareto-dominate group-specific schemes. The reason is simple: With two identical and less than perfectly correlated assets, the variance of the “extreme” portfolios $(0, 1)$ and $(1, 0)$ always exceeds that of any convex combination $(a, 1 - a)$. In terms of pension portfolios this means that if, rather than each group only providing PAYG pension via their own optimal scheme, they would mix, on a 50-50-basis the two schemes, they will experience the same (expected) levels of consumption during working age and old-age as before, but at a lower risk in old-age consumption. Taking into account, that in addition to that utility improvement, the uniform scheme will be chosen optimally (and will typically differ from the group-specific ones$^{12}$), the claim then follows.

- A bit more surprisingly (perhaps), also the converse may happen. Consider the following scenario: The two PAYG schemes have identical variances ($\sigma_1 = \sigma_2$), but differ in sign with respect to their correlation with the returns to savings: $\sigma_{R1} > 0 >$$

$^{12}$E.g., if the two group-schemes are uncorrelated ($\sigma_{12} = 0$), then it is apparent from comparing (10) and (11) that the optimal uniform scheme will be larger than the optimal separate schemes: $b^* > b^j^*$. 

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Moreover assume that $\sigma_{R2} < -\sigma_{R1}$, i.e., the correlation between the returns on saving and on scheme 2 is, in absolute terms, lower than that between the returns on saving and scheme 1. The implications of this are as follows:

- From (10), the optimal group-specific decision for group 1 would be not to have a PAYG scheme ($b_{1*} = 0$) while group 2 would run one ($b_{2*} > 0$).
- From (11), the optimal joint decision for both groups would be not to have a PAYG scheme at all ($b^* = 0$).

Group 1, thus, would be indifferent between the group-specific and the uniform solution. Group 2’s utility has been deteriorated: By choosing $b_{2*} > 0$ in the group-specific setting they revealed that their utility at this PAYG scheme is higher than at $b_2 = 0$ which is the same utility level at a uniform scheme with $b = 0$. Hence, in that scenario, the group-specific solution dominates the uniform scheme.

One might perhaps attribute that result to the absence of a PAYG scheme in uniform scenario. This is, however, not the case. Consider the same situation as previously but assume that $\sigma_{R2} > -\sigma_{R1}$, implying that it is now optimal in the uniform set-up to run a PAYG-scheme: $b^* > 0$. It is a simple task to find parameter values such that both groups suffer from moving from their optimal group-specific schemes to an optimal uniform one.

**Numerical example:** Let $\mu_R = 4$, $w = \gamma = 1 = \sigma_R = \sigma_1 = \sigma_2 = 1$, $\sigma_{R1} = 0.15$, $\sigma_{R2} = -0.3$, and $\sigma_{12} = -0.75$. It can then be calculated that: $b_{1*} = 0$ and $V_{G1}^2 = 3.25$; $b_{2*} = 0.4945$ and $V_{G2}^2 = 3.4725$; but: $b^* = 0.942$ and $V_{U1}^1 = V_{U2}^2 = 3.02297$. Hence, the group-specific solution dominates the uniform one.

This example conveys another important lesson: The optimal uniform scheme is not something like an “average” of, or a “compromise” between, the optimal group-specific schemes; it may well be entirely different. In the example, this is driven by the very high negative correlation $\sigma_{12}$ between the two PAYG schemes which makes “intra-PAYG insurance” a driving motive in the determination of the optimal joint scheme.

- From the above it should be obvious that it is also possible to construct examples where in a uniform scheme one group loses while the other benefits, relative to an
optimal selection of group-specific schemes. We refrain from providing any specific example. Observe, however, that in such a situation it is \textit{a priori} unclear which outcome will prevail.

- A uniform pension will not form voluntarily and will not be stable whenever groups are free to set up their own pension schemes and no coercion to join the uniform scheme can be exercised. If this is the case, only group-specific pension schemes can survive.\textsuperscript{13}

- A uniform pension scheme might be a stable option if its introduction can be accompanied by compensatory side payments from the winning to the losing group, i.e., if efficiency gains can be freely distributed across individuals. In a strict sense, this means a “jump out of the model” since we defined a uniform pension scheme as one where the same set of rules applies to every member – which excludes cross-subsidization.

- Given the feasibility of transfers, an obvious necessary condition for compensatory payments to support a uniform scheme is that the winners’ gains do indeed exceed the losers’ losses. But this need not be the case:

  \textbf{Numerical example:} Consider the same data as in the previous example but change \(\sigma_{12} = -0.75\) into \(\sigma_{12} = 0\). As before, we have \(b_{1}^{*} = 0\), \(V_{1}^{U} = 3.25\), \(b_{2}^{*} = 0.4945\), and \(V_{2}^{U} = 3.4725\). But the uniform solution changes into \(b^{*} = 0.2275\) with \(V_{1}^{U} = V_{2}^{U} = 3.2756\). Given our preference specification, utility differences can be readily interpreted as differences in (money-valued) consumption levels. With that interpretation it is clear that upon a transition from group-specific to a uniform scheme the winning group 1 cannot compensate the losing group 2.

Within the specific framework of this section, can any “general” condition be given such that a uniform scheme is Pareto-preferred to a selection of group-specific ones? The following result provides an answer:

\textsuperscript{13}In terms of cooperative game theory, a uniform pension scheme is not in the core of the coalitional pension game. Dealing with the question whether insurers with different risk characteristics should establish a mutual insurance scheme or rather stay single, a corresponding observation was already made in Borch (1962).
Fact 1 A uniform scheme with $b^* > 0$ is Pareto-better than group-specific schemes iff and only iff:

\[
\sigma_{R1} + \sigma_{R2} \ < \ 0 \\
2\sigma_{12}\sigma_{Ri}^2 - 2\sigma_{R1}\sigma_{R2}\sigma_{i}^2 - \sigma_{R1}^2\sigma_{i}^2 + \sigma_{R2}^2\sigma_{j}^2 \ < \ 0 \quad \text{for} \ i = 1, 2; \ i \neq j.
\]  

(13a) \hspace{2cm} (13b)

Proof: Recall that (13a) is the (necessary and sufficient) condition such that $b^* > 0$ if a uniform scheme is preferred. Conditions (13b) emerge, for each $i$, from a utility comparison of $V^i_G$ and $V^i_U$. ■

The conditions provided in Fact 1 are hardly accessible.\textsuperscript{14} Observe, however, that conditions (13b) together imply that

\[
\frac{\sigma_{12}}{\sigma_{1}^2 + \sigma_{2}^2} \ < \ \frac{\sigma_{R1}\sigma_{R2}}{\sigma_{R1}^2 + \sigma_{R2}^2}
\]

must hold – which is far easier to check.

4.3.5 Extensions

If individual preferences are dissimilar, it is not clear how a uniform pension scheme will look like. In addition to preferences and stochastic characteristics, the outcome of the pensions game will depend on the social choice rule or political decision mechanism that is applied. Nothing much of general interest can be said about this.

In the example we so far assumed that both groups’ separate pension schemes yield the same expected rate of return $\mu_1 = \mu_2$. Naturally, if the two group-specific schemes have unequal rates of return, a merger of the schemes would, in terms of expected old-age consumption, benefit the lower-return group and harm the higher-return group. This does not imply, however, that the former group would always welcome a merger while the latter would always oppose it; also the second moments of the distribution matter. Examples with any outcome can be easily fabricated. This rather simple observation has some relevance for current debates on broadening the membership base in, say, the German

\textsuperscript{14}Observe that the simple case of identically distributed pension returns (i.e., $\sigma_1 = \sigma_2$, $\sigma_{R1} = \sigma_{R2}$) is neatly covered by Fact 1. Here (13b) boils down to $\sigma_{12} < \sigma_1\sigma_2$ which simply conveys that the returns must be less than perfectly positively correlated. This confirms the assertion on the preferability of uniform schemes in that case mentioned above.
PAYG scheme. Currently, high incomes are under-represented in the PAYG scheme for two reasons: there is an upper ceiling on contributions and the scheme is (largely) confined to wage earners and, thus, does not comprise the above-average earnings from returns of capital or entrepreneurship. Proponents of broadening the scheme argue that including these groups will stabilize PAYG finances and allow for a greater deal of redistribution. This argument is, as the previous remarks indicate, based on first-moment considerations only. Taking into account the generic higher volatility of higher incomes might weaken (if not reverse) the argument for an expansion of the membership base to “capitalists” and entrepreneurs.

5 Conclusions

What can we learn out of these small examples?

- There is no unique answer to the question uniform vs. group-specific pension schemes. Depending on the circumstances, anything goes.

- In particular, the answer depends on the perspective under which the question is posed: In Example 1, we asked how a certain efficient benchmark situation (production efficiency) can best be realized. The pension scheme (whether uniform or group-specific) would then be chosen by a social planner. In Example 2 (at least for the case of identical preferences) the pension scheme within each category would be chosen as to maximise the utility of the members of the scheme.

- Moreover, the possibility of individuals to choose one’s type impacts on the answer: In Example 1, individuals could choose whether to be of type 1 or type 2 — and this choice imposes a constraint (or incentive compatibility condition) for group-specific pension design.

- Whether uniform schemes perform better than group-specific pension schemes depends on the effects of aggregation in a uniform scheme: In Example 1, a uniform scheme was equivalent to two separate but identical “group-specific” schemes. Example 2 showed that this not necessarily the case in general. Harmonization and uniformity are different issues.

The question of uniformity vs. group-specificity is underlying quite a number of the recent policy debates in the area of pay-as-you-go pensions schemes — but also in other
fields of social policy such as the areas of health insurance and – under a slightly different perspective – the basic income/flat tax debate. In the pension arena, think, e.g., of the German debate on uniform social insurances for old-age incomes or for health risks, the recent 2004 pension reform in Austria (“pension harmonization”), or the debate on the future of national PAYG pension schemes in an integrated European Union with mobility of factors and goods.

In theoretical terms, the question of uniform vs. group-specific pensions relates to a more general problem in coalitional game theory (for surveys see, e.g. Greenberg, 1994; Demange, 2004): the question why, in a multi-agent context, individuals do, or should, form small coalitions or even stay single rather than coalesce as one large group consisting of the entire society (the “grand coalition”). Why would society (optimally) run coalitional, group-specific pension schemes rather than one uniform, “grand coalition” pension scheme. Coalitional game theory investigates into efficient coalition structures (somewhat loosely, partitions of the population such that no other partition exists where the outcome Pareto-dominates the outcome in the original partition). In general, the number of coalitions in an efficient coalition structure varies with the underlying game. In our context the question of greatest interest is whether the “grand coalition”, i.e., a uniform pension scheme for the whole society, is efficient. Such games are called universally efficient.

More policy-oriented circles seem to have a certain preference bias towards uniform pension schemes. The most popular arguments to support this are, however, related to equality: Only in a uniform scheme, it is argued, can one ensure that everybody gets the same money’s worth in terms of pension benefits for the amount of contributions that he or she made. A second, often-heard argument — especially in countries like Germany, Austria, and France where some groups of high-income earners or some income categories are either exempt from membership in the PAYG scheme or covered by separate (and allegedly “privileged”) schemes — is that introducing a uniform scheme would broaden the tax base, thereby reduce the financing load that so far has been one-sidedly burdened on the factor labor and allow for more redistribution within the scheme. Finally, an obvious argument (that might be empirically more relevant for health insurance than for old-age provision) is that a uniform scheme would economize on administrative costs.

While (almost) none of these arguments can be easily dismissed with, our very tentative analysis suggests that the debate might also take into account the following aspects:

- Pension games are not necessarily superadditive in the sense that uniform
[larger] pension schemes allow for Pareto-improvements over group-specific [smaller] schemes.

- Uniform schemes may not necessarily be stable in the sense that groups captured by the scheme would not individually (or in smaller coalitions) wish to open their separate schemes.

- Even within a “uniform scheme” it might be desirable – in order to reap overall potential efficiency gains – to award privileges (“side payments”) to some of the participating groups.

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References


Groezen, B. van, A.C. Meijdam and H. Verbon, 2002, Social Security Reform and Popu-
lation Ageing in a Two-Sector Growth Model, CentER Discussion Paper No. 2002- 25,
Tilburg University.

Guesnerie, R., and C. Oddou, 1987, Increasing Returns to Size and Their Limits. Scandi-

Hauenschild, N., 1999, Uncertain Incomes, Pay-As-You-Go-Systems, and Diversification,
Zeitschrift für Wirtschafts- und Sozialwissenschaften 119, 491–507.

Theurle et al. (eds.), Kompendium der sterreichischen Finanzpolitik. Springer: Vienna etc. 507-543.

and Theoretical Economics 146, 640-647.

Homburg, S., and W.F. Richter, 1993, Harmonizing Public Debt and Pension Schemes in
the European Community. Journal of Economics 7 (Supplementum), 51-63.

ISSA – International Social Security Association, 2000, Social Security Programs

Ruland (eds.), Handbuch zur Altersversorgung. Fritz Knapp Verlag, Frankfurt/Main,
pp. 151–170.

L’Observatoire des Retraites (eds.), 2003, La Retraite au fil de l’histoire. 
http://www.observatoire-retraites.org/observatoire/rubriques/dossiers/.

Merton, R.S., 1983, On the Role of Social Security as a Means for Efficient Risk Sharing in
an Economy where Human Capital is Not Tradable. In: Bodie, Z, and J.B. Shoven (eds.),
Financial Aspects of the United States Pension System. The University of Chicago


American Economic Review 91, 596-610.


