Social Security Provisions and Entrepreneurial Risk Taking

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Abstract: We investigate the effects of social security provisions on entrepreneurial activities, measured by the amount of wealth that entrepreneurs are willing to invest in their businesses. (i) If the pension scheme only has lump-sum components and there is no specific old-age uncertainty, entrepreneurial activities will increase if and only if the pension scheme is actuarially fair. (ii) If the pension scheme only has lump-sum components but if specific old-age uncertainty prevails, entrepreneurial activities might increase even if the pension scheme is less than actuarially fair. (iii) Bismarckian pension schemes marginally distort the incentives to invest. (iv) If the entrepreneur partially finances old-age consumption by selling his business, the pension scheme may boost investment even if it is actuarially less than fair.

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1 Introduction

For market-based economies, entrepreneurs and self-employed play a pivotal role in creating value-added. Their activities are crucial for economic growth and prosperity, and many governments acknowledge this by actively encouraging the creation of small businesses and new entreprises and the risky decision to become self-employed (Dutz et al., 2000).[1] Yet, apart from these promotional efforts, there are other and more substantial governmental activities whose impact on entrepreneurship might *grosso modo* be less beneficial: taxation (Kanbur, 1981; Cullen and Gordon, 2002), regulations, costly measures of labour protection (Blau, 1987; Kanniainen and Vesala, 2000) and others.[2]

In all developed countries, social policy and welfare state provisions form substantial parts of government activities. Given their tax- or transfer-like nature, many of these provisions can be expected to affect entrepreneurial activities. At least three different channels are conceivable: First, welfare state measures may directly impact on occupational choices, i.e., on the decision to become self-employed or not. In many countries, welfare policies discriminate between employed workers and entrepreneurs. The self-employed are generally given a lower degree of social protection or, seen from a different angle, a higher degree of economic freedom than employees; this holds for unemployment benefits, health insurance, and old-age provisions. Ilmakunnas and Kanniainen (2001) theoretically and empirically show that the larger the discrimination against entrepreneurs, relative to normal workers, in welfare regulations is, the lower is the rate of entrepreneurship. For the case of old-age security, Steinberger (2005) shows that entrepreneurship is lower the more generous is the pension scheme to employed workers; entrepreneurs in that model are excluded from social security. A similar finding is obtained in a general equilibrium analysis of entrepreneurship by Wagener (2000).

Second, welfare state measures may impact on hiring decisions or, more general, input choices made by entrepreneurs even if entrepreneurs themselves are not among their beneficiaries. In many countries, social insurance for unemployment, health, or old-age incomes is co-financed by employers and employees; however, only employees are eligible for benefits. The incidence

[1] Theoretically, the case for promoting entrepreneurship is unclear though: Kanbur (1981) shows that Knightian entrepreneurship might be in an inefficiently large supply in a laissez-faire economy. Black and de Meza (1997), argueing that this finding is due to particular assumptions in Kanbur's model, propose a different approach that yields underprovision of entrepreneurship in a laissez-faire world.

[2] In quite many instances, the impact of institutional factors or government activities on entrepreneurship is unclear both theoretically and empirically (see Blanchflower, 2000, for a survey).

of such co-financing is generally unclear. However, the sheer volume of such payments makes it very likely that they will impact on entrepreneurial factor demand.

Third, welfare state measures directly targeted at entrepreneurs also impact on their activities. Most of the (scant) literature into that direction investigates into entrepreneurial risk-taking which, in the Knightian view on entrepreneurship, is the feature that distinguishes entrepreneurs from other agents in the economy: an entrepreneur's profit income is unavoidably riskier than the wage income of an employed person. Domar and Musgrave (1944) argue that proportional taxation of entrepreneurial profits with full loss offsets promotes risk-taking. Interpreting this loss-offset as a rudimentary social security net for entrepreneurs, Domar and Musgrave (1944) thus find a positive effect of the welfare state on entrepreneurship and risk-taking. Sinn (1995, 1996) generally argues that measures of the welfare state may lead to higher risk-taking; Bird (2001) finds supporting empirical evidence for this hypothesis.

In this paper we focus on the impact of social security provisions (i.e., public pension schemes) on entrepreneurial risk-taking. For the self-employed, precautions for old-age are subject to additional risks, compared to normal employees. Entrepreneurs face a higher risk exposure when saving for old-age consumption: the income risk itself (having to save out of risky incomes) and the return risk (earning a random return on individual old-age saving). Quadrini (2000) and Heaton and Lucas (2000) demonstrate that entrepreneurs, by virtue of a higher degree of risk-taking in their active life, save more and more conservatively than the rest of the population. From this one might conclude that entrepreneurs would especially welcome the beneficial insurance or diversification features for their old-age consumption which (typically relatively riskless) social security schemes provide. Yet, in many countries, entrepreneurs are not (compulsory) members of social security schemes. As shown in Wagener (2000), pension provisions specifically targeted at entrepreneurs might actually *reduce* entrepreneurship in a general equilibrium. Moreover, this paper does not find convincing support for the view that entrepreneurs need social security less urgent than normal workers.

The present paper is less ambitious: It simply analyses the effect of a compulsory membership in a pension scheme that charges contributions during active period and distributes benefits during old-age on entrepreneurial activities and, in particular, on the amount of wealth that an entrepreneur is willing to invest into his risky business. We identify three main channels, which are in turn discussed in Sections 3 to 5:

Income effects: Suppose that the pension scheme only entails lump-sum components and that there is no uncertainty (about old-age consumption or whatever) beyond that stemming from risky investment in the active period. Depending on whether the present value of pension benefits minus contributions is positive or negative, the pension scheme implies an increase or a reduction in lifetime income, irrespective of the entrepreneur's investment. As attitudes to risk-taking vary with the wealth position and, in particular, with decreasing absolute risk aversion wealthier people are more willing to take risks, an actuarially less than fair pension scheme depresses entrepreneurial investment while an actuarially more than fair one boosts investment.[3]

Insurance effects: Again assume that the pension scheme only entails deterministic and lump-sum components. But now assume that private old-age provisions (savings) are subject to a return risk. An actuarially more [less] than fair pension scheme then increases [decreases] the *expected* present value of life-time income. However, the pension scheme also serves as an insurance device since it allows to risklessly shift consumption between active period and old age. For entrepreneurial investment, the risk of old-age saving constitutes a background risk, and the lump-sum pension scheme allows some immunisation against that risk. Under plausible assumptions, reducing background risks *increases* the incentives to primary risk-taking. Clearly, income effects (see the previous item) must also be taken in account here. However, due to its insurance (or risk-reduction) effect, the pension scheme raises the incentives for entrepreneurial investment when the scheme is only actuarially fair and even if it is actuarially less than fair (but clearly not too much so).

Distortionary effects: Typically, pension schemes levy income-dependent (for entrepreneurs: profit-dependent) contributions and, in Bismarckian schemes, also pay out income-related benefits. Hence, the pension scheme may distort the marginal incentives to generate income, i.e., to invest in the business. However, the direction of this distortion is *a priori* unclear. Among others, it depends on the whether the entrepreneur can finance his old-age consumption from selling his business. For many groups of self-employed (such as farmers, owners of small manufacturing firms, lawyers) the revenues from selling the farm, firm or office make up for a considerable share of their financial needs in old-age. When deciding on how much to invest in his busi-

[3] Some caveats concerning repercussion effects of old-age provisions through private savings have to be added here.

ness, the entrepreneur will take into accounts the (typically non-negative) resale value of his enterprise. Contributions to the pension scheme and, in Bismarckian schemes, also pensions depend on current income (profits) during the active period only and do, thus, not capture the prospects or proceeds from selling the business upon entry into retirement. Hence, a purely profit-related pension scheme indeed marginally distorts the incentives to invest — but in a possibly unexpected way: In a Bismarckian scheme, one additional euro of investment creates as an additional social security tax burden the tax rate (τ, say) multiplied with marginal profits. Additional contributions generate additional pension claims whose present value is also proportional to marginal profits (with a factor of α, say). Then the marginal distortion of the pension scheme is given by $(\alpha - \tau)$ times marginal profits. On average, marginal profits are positive under uncertainty, such that the distortion goes into the direction of the sign of the return differential $(\alpha - \tau)$ – which confirms a first-order intuition that a "profitable" pension scheme (with $\alpha > \tau$) raises investment. However, if the investor also takes into account the resale value of the estate, his decision will be governed by marginal profits *plus* the marginal resale value. As the latter are strictly positive, the optimal investment level might be reached at a point where average marginal profits are *negative*. Combining this with the previous arguments, a "profitable" pension scheme (with $\alpha > \tau$) actually may *depress* investment. Put differently: If the government wants to promote investment for entrepreneurs with high resale options for their business, it should force entrepreneurs into a less than fair pension scheme.

2 A Simple Model for Entrepreneurial Investment

2.1 Set-up

Consider the following two-period life-cycle model of an entrepreneur's biography: In period 1, the entrepreneur invests in his business and saves for old-age. In period 2, he retires from his business, possibly sells his property, and lives on the proceeds of his saving, on the revenues earned when selling his estate, and possibly on pensions. His consumption levels c_1 and c_2 during the two periods of his life are given by

$$c_1 = \pi(I, \theta_1) - s - T; \tag{1a}$$

$$c_2 = R(\theta_2) \cdot s + V(I, \theta_2) + P. \tag{1b}$$

Here, I denotes business investment which, together with some random variable θ_1, determine profits π. Saving is denoted by s. Taxes T and pensions P will be explained below. Variable V denotes the resale value of the entrepreneur's business at date 2; it is determined by the size of investment I and a random parameter θ_2 that characterizes the economic environment at date 2.[4] This parameter also determines the returns on saving, given by the interest factor R.

The entrepreneur derives utility from consumption in the two periods of his life. His von-Neumann/Morgenstern utility function is additively time-separable:

$$U = \mathbf{E}_{\theta_1} \left\{ u(c_1) + \mathbf{E}_{\theta_2} \hat{u}(c_2) \right\} \tag{2}$$

The entrepreneur's problem then is to choose I and s such that lifetime-expected utility is maximized. Given the time structure of stochastic events, this can be written as

$$\max_I \mathbf{E}_{\theta_1} \left\{ \max_s \left[u(c_1) + \mathbf{E}_{\theta_2} \hat{u}(c_2) \right] \right\}. \tag{3}$$

We impose several (and, to a variable degree, restrictive) assumptions on the primitives of our model:

Random variables. The two primitive random variables in our model, θ_1 and θ_2 are integrable and stochastically independent. Each θ_k takes values from a closed interval Θ_k of the positive real line. We denote the densities by $\phi_k = \phi_k(\theta_k)$. We assume that the interest factor R is a strictly increasing function of θ_2: $R'(\theta_2) > 0$.

Profit function. An entrepreneur's profits π accrue in period 1 and depend on investments $I \in \mathbb{R}_+$ and the random effect θ_1. The decision on I has to be made before the resolution of θ_1.

[4] The resale value in period 2 does thus not depend on realized past profits π. This simplifying assumption is *not* innocuous with respect to our results. However, having V also depend on π leads to unaccessible formal expressions.

Denoting partial derivatives by subscripts, we assume that the profit function satisfies:[5]

$$\pi(0,\theta_1) = 0 \tag{4a}$$

$$\pi_I(0,\theta_1) > 0 \tag{4b}$$

$$\pi_{II}(I,\theta_1) < 0 \tag{4c}$$

$$\pi_{\theta_1}(I,\theta_1) > 0 \tag{4d}$$

$$\pi_{I\theta_1}(I,\theta_1) > 0 \tag{4e}$$

for all $\theta_1 \in \Theta_1$ in items (a) and (b) and all $(I, \theta_1) \in \mathbb{R}_+ \times \Theta_1$ in the other items. Assumption (a) could be easily changed into $\pi(0, \theta_1) = \bar{W} > 0$, thereby capturing that the entrepreneur owns some initial wealth $\bar{W}$ that remains unchanged if he decides not to invest at all in a business. By items (d) and (e), θ_1 can be called a positive shock: it affects both profits and marginal profits positively.

Utility function. For $\nu = u, \hat{u}$ we assume that $\nu'(c) > 0 > \nu''(c)$ for all $c > 0$ and that the Inada limit conditions hold. Further we assume decreasing absolute risk aversion (DARA),

$$\left(-\frac{\nu''(c)}{\nu'(c)}\right)' < 0. \tag{5}$$

We write

$$\mathcal{R}_\nu(c) := -c \cdot \frac{\nu''(c)}{\nu'(c)} \tag{6}$$

for the Arrow-Pratt index of relative aversion for the utility function ν.

Resale Value. The function V represents the money value at which the entrepreneur can sell his firm in period 2. This value may depend on I (i.e., on the investment undertaken to build up the enterprise) and on a random event θ_2. We assume that it does not depend on past profits – which potential buyers might use as an indicator for the future profitability of the business. Hence, V is also independent of the random parameter θ_1. We assume that the resale value of

[5] As an example, think of a firm that produces a numéraire output using a standard Inada technology $\theta_1 \cdot f(I)$ where investment generates convex and non-stochastic costs $C(I)$. Then profits $\pi = \theta_1 \cdot f(I) - C(I)$ satisfy all five assumptions.

the business is never negative, does not decrease in I, and is concave in I:

$$\begin{aligned} V(I,\theta_2) &\geq 0; && \text{(7a)} \\ V_I(I,\theta_2) &\geq 0; && \text{(7b)} \\ V_{II}(I,\theta_2) &\leq 0 && \text{(7c)} \end{aligned}$$

for all $(I,\theta_2) \in \mathbb{R}_+ \times \Theta_2$. In each case, the equality sign might hold. E.g., the case $V_I \equiv 0$ (the resale value of the business does not depend on I) might be due to the fact that investments are not verifiable (even not *ex post*). Furthermore, we do not make any assumption about the direction in which V depends on θ_2.

Pension System. The pension system is represented by a contribution function T and a pension function P. We assume that T can (only) depend on profits, $T = T(\pi)$, such that contributions might be random *ex ante*. For simplicity, we only consider linear contribution schemes:

$$T(\pi) = \bar{T} + \tau \cdot \pi \tag{8}$$

with lump-sum taxes $\bar{T} \geq 0$ and marginal contribution rate $\tau \in [0,1)$. Pensions P include lump-sum components plus possibly contribution- (and thus, via (8), profit-) dependent components. Again we only consider linear pension formulas:

$$P(T(\pi),\theta_2) = \bar{P} + A(\theta_2) \cdot T(\pi) \tag{9}$$

with $\bar{P}$ as the lump-sum part.[6] The variable

$$A(\theta_2) = \frac{\partial P}{\partial T} \geq 0 \tag{10}$$

denotes the marginal return on an additional euro of pension contributions. For PAYG schemes, A generally equals the growth rate of the social security tax base and is then called (marginal) Aaron factor (Aaron, 1966); we adopt this labelling. For $\tau > 0$ and $A > 0$ the pension scheme is Bismarckian in the sense that pensions vary positively with income during working life. For $\tau \geq 0$ and $A = 0$, the pension scheme can be called Beveridgean since it only includes a basic pension. In a Beveridgean scheme, the marginal Aaron factor is zero. The implicit rate of return of the pension scheme P/T coincides with the marginal Aaron factor if and only if $\bar{P} = A(\theta_2) \cdot \bar{T}$ almost everywhere, which includes as a special case the absence of any lump-sum components.

[6] In principle, $\bar{P}$ could vary with the state θ_2, too. This leads to complex effects which are beyond the scope of our analysis.

2.2 Optimal Decisions

2.2.1 Savings

In period 1, after investments have been sunk and profits are realized, the entrepreneur decides on his saving – which may be understood as his private old-age provisions. He determines[7]

$$\begin{aligned} S(I,T,P,\theta_1) \quad := \quad & \arg\max_S \left\{ u\left[(1-\tau)\pi(I,\theta_1) - \bar{T} - S\right] \right. \\ & \left. + \mathbf{E}_{\theta_2} \hat{u}\left[R(\theta_2)\cdot S + V(I,\theta_2) + \bar{P} + \tau A(\theta_2)\pi(I,\theta_1)\right] \right\}. \qquad (11) \end{aligned}$$

The FOC for S requires that expected and discounted marginal utilities of consumption are equalized:

$$-u'[c_1(I,\theta_1)] + \mathbf{E}_{\theta_2}\left(R(\theta_2)\cdot \hat{u}'[c_2(I,\theta_1,\theta_2)]\right) = 0. \qquad (12)$$

The comparative statics of S are straightforward (their formulas are collected in the Appendix). If there is no pension scheme or either the contribution or the benefit side of the pension scheme is lump-sum, then saving (i.e., private old-age provisions) will be higher in more favorable states of the economy (the higher θ_1). Increasing contributions and, thus, reducing disposable income in period 1 negatively impacts on private old-age provisions. An increase in lump-sum pension benefits also reduces incentive to take private old-age provisions.

2.2.2 Optimal Investment Decisions

The optimal investment decision are derived from the following program:

$$\max_I \mathbf{E}_{\theta_1}\left\{ \max_S \left[u\left(\pi(I,\theta_1) - T - S\right) + \mathbf{E}_{\theta_2}\hat{u}(RS + V(I,\theta_2) + P)\right]\right\}. \qquad (13)$$

At the optimal investment level, the expected marginal lifetime utility must be zero:

$$\mathbf{E}_{\theta_1}\left\{\pi_I(1-\tau)u'(c_1) + \mathbf{E}_{\theta_2}\left[(A(\theta_2)\tau\pi_I + V_I)\cdot \hat{u}'(c_2)\right]\right\} = 0.$$

Note that investment (possibly) affects the entrepreneur's retirement consumption via two channels: through the resale value of the firm and through the pension system if that is Bismarckian (i.e., $A(\theta_2)\tau \neq 0$ for at least some θ_2).

[7] We abuse notation when plugging the whole pension scheme P and T (rather than its primitive parameters) into the saving function. Confusion should not arise from this.

Combining this with the FOC (12) for optimal saving in any state θ_1 we obtain the following condition for optimal investment:

$$\mathbf{E}_{\theta_1}\Big\{ \underbrace{\pi_I \cdot \tau \cdot \mathbf{E}_{\theta_2}\left[(A-R)\cdot \hat{u}'(c_2)\right]}_{=:\Omega} + \underbrace{\mathbf{E}_{\theta_2}\left((\pi_I R + V_I)\cdot \hat{u}'(c_2)\right)}_{=:\Gamma} \Big\} = 0. \qquad (14)$$

We arranged this expression such that Ω contains all marginal effects of investment that are generated through the pension scheme while Γ comprises the non-pension effects. As the objective function in (13) is strictly concave in I,[8] we can say that the pension scheme *marginally encourages [discourages]* entrepreneurial investment if

$$\mathbf{E}_{\theta_1}\Omega \; > \; [<] \;\; 0. \qquad (15)$$

If $\mathbf{E}_{\theta_1}\Omega = 0$, we say that the pension scheme is non-distortionary. As a first and immediate observation we obtain from (14):

Fact 1 *Marginal incentives to invest are not distorted by the pension scheme (i.e., $\Omega \equiv 0$) if*

- *pension contributions are lump-sum: $\tau = 0$; or if*
- *the pension scheme is actuarially fair: $A(\theta_2) = R(\theta_2)$ a.e.-θ_2.*

The second item of Fact 1 identifies a funded pension scheme (where contributions earn the interest rate) as neutral with respect to entrepreneurial investment I. If the conditions of Fact 1 are not met, the analysis gets quite involved. Being an integral over a product of functions, the expression $\mathbf{E}_{\theta_1}\Omega$ is generally inaccessible without imposing further restrictions. In particular, knowledge of the marginal return differential $(R(\theta_2) - A(\theta_2))$ and of the sign of $\mathbf{E}_{\theta_1}\pi_I$ or $\mathbf{E}_{\theta_1}(\pi_I \mathbf{E}_{\theta_2}\hat{u}')$ are necessary.

At this place, the crucial role of the resale value V becomes obvious: Without any resale value ($V = V_I = 0$ for all I) and in the absence of a pension scheme, the expected marginal utility from investment is zero: $\mathbf{E}_{\theta_1}(\pi_I \mathbf{E}_{\theta_2}\hat{u}') = 0$.[9] Hence, one could – in a first approximation – presume that the direction of the marginal distortion of investment is determined by the sign of the marginal return differential $(A - R)$ between the pension scheme and private saving.[10] If, however, there is a positive marginal resale value $V_I > 0$, we get that $\mathbf{E}_{\theta_1}(\pi_I \mathbf{E}_{\theta_2}\hat{u}') < 0$ in the

[8]The second-order derivative of (13) with respect to I is always negative if V is concave in I.

[9]By the concavity of $\hat{u}$, this implies that expected marginal profits are positive: $\mathbf{E}_{\theta_1}\pi_I > 0$.

[10]In the introduction, we tentatively argued with a return differential $(\tau - \alpha)$. Putting $\alpha = \tau A/R$ yields the analogy to the discussion here.

absence of a pension scheme: the expected marginal utility from direct profits out of investment is negative. Hence, applying the same first-order approximation as before, the direction of the marginal distortion of investment is opposite to the marginal return differential between the pension scheme and private saving!

We will return to this issue in Section 5 where we will show for an example that introducing an actuarially less than fair scheme (i.e., $A < R$) may boost investment if old-age consumption is partly financed from the resale revenues of the business estate. The subsequent sections will, however, discuss the case of lump-sum pension schemes (that have $\mathbf{E}_{\theta_1}\Omega = 0$ by definition). While such schemes do not affect the marginal returns on investment, they nevertheless exert income and insurance effects. We go through several cases.

3 Lump-Sum Pensions without Uncertainty in the Second Period

Let us first consider the case where uncertainty only affects profits, but not second-period consumption. I.e., θ_2 is deterministic and all integrals with respect to θ_2 can be omitted from the previous formulae.

Assume that the pension formula only contains lump-sum components: $T = \bar{T}$ and $P = \bar{P}$. The FOC (14) then simplifies to

$$\mathbf{E}_{\theta_1}\left\{(\pi_I R + V_I)\cdot \hat{u}'(c_2)\right\} = 0. \tag{16}$$

Note that c_2 is uncertain since it depends on θ_1 via saving. Indeed, the properties of the saving function will play a crucial role for our analysis.

The saving function is *convex* if the marginal rate of saving – which is endogenous here – is weakly increasing in first-period income and weakly decreasing in second-period income. I.e., given a saving problem $\max_S\{u(Y_1 - S) + \mathbf{E}_{\theta_2}\hat{u}(R(\theta_2)S + Y_2)\}$ with solution $S = S(Y_1, Y_2)$ a convex saving function satisfies:

$$S_{Y_1Y_1}(Y_1, Y_2) \geq 0 \quad \text{and} \quad S_{Y_1Y_2}(Y_1, Y_2) \leq 0. \tag{17}$$

This assumption will – if R is non-stochastic – be satisfied if and only if utilities exhibit convex absolute risk tolerance (where risk tolerance is the inverse of the Arrow-Pratt index of absolute risk aversion). Equality in (17) will hold when $u, \hat{u}$ belong to the HARA class of utility functions which have linear risk tolerance. If R is random, the requirements on u and $\hat{u}$ are more complex, but need not bother us here (see Carroll and Kimball, 1996). Convexity of the saving function (or, equivalently, the concavity of the consumption function) is an often made and empirically justified assumption whose theoretic roots at least date back to Keynes (for details see Gollier, 2001, ch. 15).

Fact 2 *Suppose that the saving function is convex. Then:*

- *Entrepreneurial investment is increasing in pensions:* $\dfrac{\partial I}{\partial \bar{P}} > 0$.
- *Entrepreneurial investment is decreasing in contributions:* $\dfrac{\partial I}{\partial \bar{T}} < 0$.

From Fact 2, increasing the pension boosts entrepreneurial investment while an increase in taxes depresses it. This sounds natural, but does not yet tell us anything about the effects of introducing a pension scheme that consists of both contributions and pensions. The question we are interested in reads: What relationship between contributions and pensions must exist such that entrepreneurial investment is increased [decreased; remains constant]? Here is a partial answer:

Fact 3 *Consider a pension reform, i.e., a change* $(\mathrm{d}\bar{T}, \mathrm{d}\bar{P}) > (0, 0)$ *in the lump-sum pension formula.*

- *Suppose that the pension reform does not increase the marginal rate of saving. Then entrepreneurial investment is increasing if, but not only if, a change in lump-sum contributions and benefits satisfies* $\mathrm{d}\bar{P} \geq R \cdot \mathrm{d}\bar{T}$.
- *Suppose that utility functions satisfy* $\hat{u}(c) = \delta \cdot u(c)$ *for some positive constant* δ. *Then investment is non-increasing if and only if a change in lump-sum contributions and benefits satisfies* $\mathrm{d}\bar{P} \leq R \cdot \mathrm{d}\bar{T}$.

From Fact 3 we learn that a lump-sum pension scheme that is more [less] profitable than private saving in capital markets increases [depresses] entrepreneurial investment. The intuition is quite clear: The pension scheme makes the entrepreneur wealthier [poorer] in terms of the present value of life-time income whenever it is more [less] profitable than saving. Decreasing absolute risk aversion (DARA), however, implies that the readiness to risk-taking positively varies with the wealth position. Thus, an actuarially more [less] than fair pension scheme increases [reduces] risky entrepreneurial investment.[11]

Given the simplicity of this DARA-argument one might wonder why Fact 3 does not show this all too clearly. The reason is a second-order effect via the marginal rate on saving.[12] In the standard form of additively separable utilities with a constant rate of time preference, $U = u(c_1) + \delta u(c_2)$, such an effect cannot occur, however. There, the marginal rate of saving remains unaffected from lump-sum pension reforms and, thus, the DARA-argument outlined above goes through unabatedly.

4 Lump-Sum Pensions with Uncertainty in the Second Period

Let us re-introduce uncertainty in the second period, represented by θ_2. Now, the analysis of a lump-sum pension scheme gets quite complicated. We proceed with the following simplifying assumptions:

(A1) Utility of old-age consumption is logarithmic: $\mathcal{R}_{\hat{u}} = 1$.

(A2) Utility of consumption in the active period is logarithmic: $\mathcal{R}_u = 1$.

(A3) Initially, there is no pension scheme: $\bar{P} = 0$ or, equivalently, $c_2 = R \cdot S + V$.

(A4) The resale value of the estate is proportional to the interest factor: $V(I, \theta_2) = \psi(I) \cdot R(\theta_2)$ for some weakly increasing function $\psi(I)$.

Assumptions (A1) and (A2) together are a special case of the assumptions in Fact 3. Logarithmic utility functions belong to the HARA class that induces convex saving functions. Moreover, the saving function is linear (we will come back to this below). By (A3), we focus on the introduction of a *new* pension scheme. Assumption (A4) means that resale value and interest factor are perfectly correlated. However, (A4) also encompasses that there is no resale value at all (i.e., $\psi \equiv 0$) or that it does not depend on I (i.e., $\psi = \bar{\psi}$). We first obtain a counterpart to Fact 2:

Fact 4 *Suppose that the saving function is convex and that (A2) to (A4) hold. Then:*

[11] This also explains Fact 2. Formally, the role of DARA becomes visible in the function H_3 and its counterparts in the proof of Fact 2 which contain the (negative of the) Arrow-Pratt measure of absolute risk aversion.

[12] Formally, the cofactor Γ in H_3, which represents the change in saving that is induced by the pension reform, may turn an otherwise decreasing absolute risk aversion for consumption c_2 into an increasing one if Γ_{θ_1} is positive (and strongly so). Somewhat loosely, we can write H_3 as: $\mathrm{d}c_2 \cdot (\hat{u}''/\hat{u}')$. We must require that $\mathrm{d}[\mathrm{d}c_2 \cdot (\hat{u}''/\hat{u}')] = \mathrm{d}^2 c_2 \cdot (\hat{u}''/\hat{u}') + \mathrm{d}c_2 \cdot \mathrm{d}(\hat{u}''/\hat{u}') > 0$.

- *Entrepreneurial investment is increasing in pensions:* $\dfrac{\partial I}{\partial \bar{P}} > 0$.
- *Entrepreneurial investment is decreasing in contributions:* $\dfrac{\partial I}{\partial \bar{T}} < 0$.

The main (single and decisive) purpose of assumptions (A2) to (A4) in Fact 4 is to neutralize the effect of π_I in the determination of g_{θ_2} in (29). This is important as the sign of marginal profits π_I is unclear (it depends on the state of Nature θ_1 and on I). Information about the sign of g_{θ_2} is essential, however, to utilize Chebyshev's Inequality in the proof of Fact 4.

Put differently: Without assumptions (A2) to (A4) (or any single of them) the reactions of entrepreneurial investment upon changes in pension parameters $\bar{T}$ and $\bar{P}$ could well be different than indicated by Fact 4. Generally, these reactions would not have unambiguous directions.

Fact 4 does not yet tell us much about the effects of an entire pension scheme. Searching for an analogue to Fact 3, we can at least offer a partial one:[13]

Fact 5 *Suppose that (A1) to (A4) hold. Then entrepreneurial investment is non-decreasing if and only if a change in lump-sum pensions satisfies* $\mathrm{d}\bar{T}/\mathrm{d}\bar{P} \leq \mathbf{E}_{\theta_2}(R^{-1})$.

Note that, by Jensen's Inequality,

$$(\mathbf{E}_{\theta_2} R^{-1})^{-1} < \mathbf{E}_{\theta_2} R \tag{18}$$

for all non-degenerate distributions of θ_2.[14] Compared to the case of non-stochastic old-age consumption (Fact 3), uncertainty in old age leads to lower return requirements on the pension scheme in order to increase entrepreneurial investment: To promote investment when old-age incomes are certain, the pension scheme has to offer at least the same rate of return to the contributor as do private capital markets ($\mathrm{d}\bar{P}/\mathrm{d}\bar{T} \geq R$), we now find that also pension schemes that offer lower expected rates of return to their contributors than funded private saving will be beneficial for entrepreneurial investment.

The intuition behind this observation can be based on the idea of *risk vulnerability* (Gollier and Pratt, 1996):[15] With respect to entrepreneurial investment, the risk θ_2 associated with old-age saving constitutes an (independent) background risk. If agents' preferences exhibit risk

[13] As shown in the Appendix, assumptions (A1) and (A2) imply that the saving function is linear in first-period income and that at $\bar{P} = 0$ the marginal savings rate does not change locally when $\bar{P}$ is marginally increased.

[14] For degenerate distributions, $(\mathbf{E}R^{-1})^{-1} = \mathbf{E}R = R$. Hence, Fact 5 coincides with Fact 3.

[15] Logarithmic utility functions exhibit risk vulnerability. In fact, they even show standardness (i.e., DARA and decreasing absolute prudence $(-u'''(c)/u''(c))' < 0$).

vulnerability they will react upon a new background risk with a reduction of their initial risk-taking. I.e., if one compared the scenarios of Sections 3 and 4 in the absence of any pension scheme, investment would be higher in the former.

Lump-sum pension schemes provide a partial immunisation against this background risk: Unlike risky saving, they allow a *riskfree* transfer of consumption from period 1 to period 2. If the expected rate of return of the pension scheme equals or even exceeds that of private saving, the pension scheme thus "costlessly" or even profitably dampens the entrepreneur's exposure to the background risk — and the risk-vulnerable entrepreneur will react with an increase in his investment. But even if the expected rate of return in the pension scheme falls short of the returns to saving, the insurance effect will raise investment, in spite of a negative income effect. Clearly, there is a limit to this insurance effect: If the rate of return in the pension scheme falls below some threshold value (in the example of Fact 5 identified as $\mathbf{E}^{-1}(R^{-1})$) the income effect (which is negative then) will become dominant.

5 Distortionary Pensions

Let us now consider the case that contributions are raised as profit taxes and that pensions are, *sensu* Bismarck, strictly contributions- and thus income-related (no lump-sum). We again assume first that there is no uncertainty about the second period (except that which will, from an *ex ante* view, be carried over from period 1). In particular, R and A are deterministic. Then the FOC for optimal investment reads as:

$$\mathbf{E}_{\theta_1}\left\{\hat{u}'(c_2)\cdot[\pi_I\cdot((1-\tau)R+\tau A)+V_I]\right\}=0. \tag{19}$$

We are interested in $\partial I/\partial\tau$, i.e., the reaction of optimal investment to a higher contribution rate and, as a consequence of the Bismarckian assumption, also to a higher *gross* replacement ratio.

As a counterpart to (A3) we introduce assumption

(A3') Initially, there is no pension scheme: $\tau = 0$ (which implies $T = P = 0$).

We then obtain

Fact 6 *Let (A1), (A2), (A3') and (A4) be satisfied.*

- *If the business has no resale value ($\psi = 0$ for all I), then introducing a small Bismarckian pension scheme is neutral with respect to investment: $\partial I/\partial\tau = 0$.*

- *If the resale value of the business positive and increasing in I, then introducing a small Bismarckian pension scheme increases investment if and only if the Aaron factor falls below the interest factor:*

$$\mathrm{sgn}\frac{\partial I}{\partial \tau} = \mathrm{sgn}(R - A). \tag{20}$$

The key element to understand Fact 6 is that the entrepreneur will optimally choose his investment level such that expected marginal utility from profits are negative ($\mathbf{E}_{\theta_2}(\pi_I \hat{u}') < 0$) if he also takes into account that he might resell his business in old-age. If there is no resale of the business, a profit-income based, Bismarckian pension scheme will be marginally non-distortionary (first item of Fact 6) – as one would expect. If, however, the business has a resale value that positively varies with investment, then a profit-based Bismarck-scheme distorts investment in the opposite direction of the return differential to saving.

One should not overstate this point. Given its restrictive assumptions, Fact 6 merely provides an example for a possibly surprising effect; it does not make any claim that this effect will always occur. In particular, even in this example the effect only could be identified for small pension schemes ($\tau = 0$ initially), but not for increases in the tax rate of a large-scale scheme. There, the negative income effects of an actuarially unfair pension scheme (similar to those found in Section 3) will dominate.

6 Conclusions

This paper deals with the effects of social security schemes on entrepreneurial investment. A potentially severe shortcoming of our analysis is that we do not ask whether entrepreneurs do at all welcome the pension schemes we impose upon them. The answer to this question is clear in the setting of Section 3: Expected lifetime utility is definitely increasing in lifetime wealth. Hence, we can say that a pension scheme raises investment if and only if it also raises the entrepreneur's lifetime utility. The answer is a bit more tricky in the setting of Section 4. Given its insurance property, the entrepreneur will certainly be willing to participate in a pension scheme that is, up to a certain extent, less than actuarially fair. It is, however, not clear whether the threshold levels for the return differential between saving and pensions below which (i) the entrepreneur reduces investment and (ii) the entrepreneurs would prefer to be exempted from the scheme do in fact coincide. Finally, the utility issue becomes more or less inaccessible in the setting of Section 5. To justify our omission of desirability considerations, note that probably most existing pension regulations for entrepreneurs are not the result of theoretical considerations; rather, they are the artefact of historical contingencies and perhaps of lobbying by professional interest groups. It might thus be interesting to simply consider the incentive effects of various such schemes – not questioning the rationale of their existence itself. Such a positive analysis might serve as a first step towards answering the normative question of what a pension scheme for entrepreneurs should optimally look like.

In spite of its shortcomings, our analysis might give rise to some tentative conclusions:

- Pension provisions for entrepreneurs in fact influence their investment decisions. Given the importance of entrepreneurial activities for economic growth and prosperity, this issue should not be ignored.

- Old-age related risks and business related risks cannot be treated separately since they interact in a rather complex way. This should be taken into account when designing blueprints for old-age provisions targeted at the self-employed.

- In particular, it is insufficient to only look at the return differentials between pensions and capital markets; possible insurance properties of pension schemes have also to be taken into account. Given the Knightian view that entrepreneurs take higher undiversifiable risks in their active life than the rest of the population, old-age income insurance might encourage Knightian risk-taking.

- Different groups of entrepreneurs also react differently on old-age provisions. We exemplified this point by comparing reactions to a Bismarckian scheme when entrepreneurs can or cannot sell their business when they retire; other examples are conceivable. This might call for a differential treatment of different groups of self-employed professionals and entrepreneurs.

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Appendix: The Proofs

Comparative statics of saving

Differentiating (12) yields:

$$\begin{aligned}
\frac{\partial S}{\partial \theta_1} &= -\frac{\pi_{\theta_1}}{N} \cdot \left[-u''(c_1)(1-\tau) + \mathbf{E}_{\theta_2}\left(\tau RA \cdot \hat{u}''(c_2)\right)\right] &\text{(21a)}\\
\frac{\partial S}{\partial \bar{T}} &= -\frac{u''(c_1)}{N} < 0; &\text{(21b)}\\
\frac{\partial S}{\partial \tau} &= -\frac{1}{N} \cdot \left[\pi \cdot u''(c_1) + \mathbf{E}_{\theta_2}(A(\theta_2)R(\theta_2)\hat{u}''(c_2))\right] < 0; &\text{(21c)}\\
\frac{\partial S}{\partial \bar{P}} &= -\frac{1}{N} \cdot \mathbf{E}_{\theta_2}\left[R(\theta_2) \cdot \hat{u}''(c_2(\cdot, \theta_2))\right] < 0 &\text{(21d)}
\end{aligned}$$

where

$$N := u''(c_1(I, \theta_1)) + \mathbf{E}_{\theta_2}\left(R^2(\theta_2) \cdot \hat{u}''[c_2(I, \theta_1, \theta_2)]\right) < 0.$$

■

Proof of Fact 2

The proof is based on a non-monotone version of Chebyshev's Inequality (see Mitrinovic et al., 1993, p. 248):

Lemma 1 *Let $H, G : [\underline{\theta}, \bar{\theta}] \to \mathbb{R}$ be integrable functions and let H be increasing. Furthermore, let $\phi : [\underline{\theta}, \bar{\theta}] \to \mathbb{R}_+$ be a density function on $[\underline{\theta}, \bar{\theta}]$. If, for all $y \in (\underline{\theta}, \bar{\theta})$,*

$$\frac{\int_{\underline{\theta}}^{y} G(\theta)\phi(\theta)\mathrm{d}\theta}{\int_{\underline{\theta}}^{y} \phi(\theta)\mathrm{d}\theta} \leq \int_{\underline{\theta}}^{\bar{\theta}} G(\theta)\phi(\theta)\mathrm{d}\theta, \tag{22}$$

then:

$$\mathbf{E}_{\theta}(G(\theta)H(\theta)) \geq \mathbf{E}_{\theta}G(\theta)\mathbf{E}_{\theta}H(\theta). \tag{23}$$

Equality in (23) will only hold if (22) holds with equality for all $x \in (\underline{\theta}, \bar{\theta})$. ◊

We proceed with the **proof** of Fact 2: Apply the implicit function theorem to (16) and expand by $\hat{u}'$ under the expectation to obtain:

$$\frac{\partial I}{\partial \bar{P}} = -\frac{1}{M} \cdot \mathbf{E}_{\theta_1}\left\{\underbrace{(\pi_I R + V_I) \cdot \hat{u}'(c_2)}_{=:G(\theta_1)} \cdot \underbrace{\left(R\frac{\partial S}{\partial \bar{P}} + 1\right) \cdot \frac{\hat{u}''(c_2)}{\hat{u}'(c_2)}}_{=:H_1(\theta_1)}\right\} \tag{24a}$$

$$\frac{\partial I}{\partial \bar{T}} = -\frac{1}{M} \cdot \mathbf{E}_{\theta_1}\left\{\underbrace{(\pi_I R + V_I) \cdot \hat{u}'(c_2)}_{=G(\theta_1)} \cdot \underbrace{R\frac{\partial S}{\partial \bar{T}} \cdot \frac{\hat{u}''(c_2)}{\hat{u}'(c_2)}}_{=:H_2(\theta_1)}\right\} \tag{24b}$$

Here, $M < 0$ is a negative cofactor, resulting from the SOC for optimal investment. To apply Lemma 1, put $\theta \equiv \theta_1$ and denote the density of θ_1 by $\phi = \phi_1(\theta_1)$.

- Verify that $G(\theta_1)$ is positive [negative; zero] if and only if $\pi_I(I, \theta_1) > [<; =]\, v_I/R$. Hence, G changes its sign once, from negative to positive. Furthermore $\mathbf{E}_{\theta_1} G(\theta_1) = 0$ from (16). Due to the single-crossing property of G, (22) is satisfied with strict inequality for all y in the support of θ_1 (recall that $\int^y \phi \mathrm{d}\theta_1 > 0$ for all y).

- Next calculate:

$$\begin{aligned} H_1'(\theta_1) &= R\frac{\partial^2 S}{\partial \bar{P} \partial \theta_1} \cdot \frac{\hat{u}''(c_2)}{\hat{u}'(c_2)} + \left(R\frac{\partial S}{\partial \bar{P}} + 1\right) \cdot \frac{\partial c_2}{\partial \theta_1} \cdot \left(\frac{\hat{u}''(c_2)}{\hat{u}'(c_2)}\right)' > 0; \\ H_2'(\theta_1) &= R\frac{\partial^2 S}{\partial \bar{T} \partial \theta_1} \cdot \frac{\hat{u}''(c_2)}{\hat{u}'(c_2)} + R\frac{\partial S}{\partial \bar{T}} \cdot \frac{\partial c_2}{\partial \theta_1} \cdot \left(\frac{\hat{u}''(c_2)}{\hat{u}'(c_2)}\right)' < 0. \end{aligned}$$

  These inequalities come from the facts that $\hat{u}''/\hat{u}' < 0 < (\hat{u}''/\hat{u}')'$ due to DARA, that $R\frac{\partial S}{\partial \bar{P}} > -1$ from (21d), that $\partial c_2/\partial \theta_1 = R \cdot \partial S/\partial \theta_1 > 0$ from (21a) under the assumptions made here and from the convexity (17) of the saving function.

Combine all this information to yield the proposition:

$$\begin{aligned} \frac{\partial I}{\partial \bar{P}} &= -M^{-1}\mathbf{E}_{\theta_1}(G \cdot H_1) > -M^{-1}\mathbf{E}_{\theta_1} G \mathbf{E}_{\theta_1} H_1 = 0; \\ \frac{\partial I}{\partial \bar{P}} &= M^{-1}\mathbf{E}_{\theta_1}(G \cdot (-H_2)) < -M^{-1}\mathbf{E}_{\theta_1} G \mathbf{E}_{\theta_1} H_2 = 0. \end{aligned}$$

■

Proof of Fact 3

Let $\mathrm{d}\bar{P} = k \cdot \mathrm{d}\bar{T} > 0$ for some positive constant k. We are interested in

$$\mathrm{d}I\big|_{\mathrm{d}\bar{P} = k\cdot \mathrm{d}\bar{T} > 0} = \mathrm{d}\bar{T} \cdot \left[-\frac{1}{M}\mathbf{E}_{\theta_1} G(\theta_1) \cdot H_3(\theta_1)\right] \tag{25}$$

where

$$H_3(\theta_1) := \underbrace{\left[R\left(k\frac{\partial S}{\partial \bar{P}} + \frac{\partial S}{\partial \bar{T}}\right) + k\right]}_{=:\Gamma(\theta_1)} \cdot \frac{\hat{u}''(c_2)}{\hat{u}'(c_2)}.$$

By the same token as in the proof of Fact 2, we get that $\mathrm{d}I$ is positive [negative; zero] in (25) if and only if H_3 is increasing [decreasing; constant] in θ_1. Calculate:

$$H_3'(\theta_1) = (R \cdot \Gamma + k) \cdot \left(\frac{\hat{u}''}{\hat{u}'}\right)' \cdot \frac{\partial c_2}{\partial \theta_1} + R \cdot \frac{\hat{u}''}{\hat{u}'} \cdot \frac{\partial \Gamma}{\partial \theta_1}.$$

Next verify from (21b) and (21d) that

$$\Gamma(\theta_1) = -R \cdot \frac{k \cdot R\hat{u}'' + u''}{R^2\hat{u}'' + u''} + k \gtrless 0 \quad \Longleftrightarrow \quad k \gtrless R.$$

Furthermore check that

$$\frac{\partial \Gamma}{\partial \theta_1} = R\left(k\frac{\partial^2 S}{\partial \bar{P}\partial \theta_1} + \frac{\partial^2 S}{\partial \bar{T}\partial \theta_1}\right), \tag{26}$$

which is non-positive when the pension reform $(\mathrm{d}\bar{T}, \mathrm{d}\bar{P})$ does not raise the marginal rate of saving. This given and for $k \geq R$, we get $\mathrm{d}I > 0$ whenever (but generally not only if) $\frac{\partial \Gamma_1}{\partial \theta_1} < 0$. This proves the first item of the assertion.

It is helpful to further investigate into (26). Use (21b) and (21d) to find after some lengthy calculation that

$$\frac{\partial \Gamma}{\partial \theta_1} = (k - R) \cdot \Omega \cdot \left[\frac{u''}{u'}\frac{\hat{u}'''}{\hat{u}''} - \frac{\hat{u}''}{\hat{u}'}\frac{u'''}{u''}\right] \tag{27}$$

where $\Omega = -N^{-3} \cdot \pi_{\theta_1} R^2 u'' \hat{u}'' u' > 0$. The sign of Γ_{θ_1} is thus unclear in general. If, e.g., u is CRRA of degree β and $\hat{u}$ is CRRA of degree γ, then $\mathrm{sgn}\Gamma_{\theta_1} = \mathrm{sgn}[(k-R)(\gamma-\beta)]$ which shows that indeed any sign of Γ_{θ_1} is possible. If, however, utilities u and $\hat{u}$ satisfy $\hat{u}(c) = \delta \cdot u(c)$ for some positive constant (and thus optimal saving satisfies $u'(c_1) = \delta \cdot R \cdot u'(c_2)$) we get that $\Gamma_{\theta_1} = 0$. The sign of H_3' is then solely determined by the sign of Γ_1 which is equal to the sign of $(k - R)$. This is the desired result in the second item. ■

Proof of Fact 4

Apply the implicit function theorem to (14) and expand by $\hat{u}'$ under the expectation to get:

$$\frac{\partial I}{\partial \bar{P}} = -\frac{1}{M} \cdot \mathbf{E}_{\theta_1}\mathbf{E}_{\theta_2}\left\{\underbrace{(\pi_I R + V_I) \cdot \hat{u}'(c_2)}_{=:g(\theta_1,\theta_2)} \cdot \underbrace{\left(R\frac{\partial S}{\partial \bar{P}} + 1\right) \cdot \frac{\hat{u}''(c_2)}{\hat{u}'(c_2)}}_{=:h_1(\theta_1,\theta_2)}\right\} \tag{28a}$$

$$\frac{\partial I}{\partial \bar{T}} = -\frac{1}{M} \cdot \mathbf{E}_{\theta_1}\mathbf{E}_{\theta_2}\left\{\underbrace{(\pi_I R + V_I) \cdot \hat{u}'(c_2)}_{=g(\theta_1,\theta_2)} \cdot \underbrace{R\frac{\partial S}{\partial \bar{T}} \cdot \frac{\hat{u}''(c_2)}{\hat{u}'(c_2)}}_{=:h_2(\theta_1,\theta_2)}\right\} \tag{28b}$$

Here, $M < 0$ is again a negative cofactor (different from the one above), resulting from the SOC.

- To start, note that g is (weakly) increasing in θ_2 under (A2), (A3), (A4):

$$
\begin{aligned}
\frac{\partial g}{\partial \theta_2} &= (\pi_I R' + V_{I\theta_2})\hat{u}' + (\pi_I R + V_I)\hat{u}''(R's + V_{\theta_2}) \\
&\overset{(A3)}{=} \pi_I R'\hat{u}' \cdot \underbrace{(1 - \mathcal{R}_{\hat{u}})}_{=0;\ (A2)} - \pi_I \hat{u}'' \cdot \underbrace{(R'V - RV_{\theta_2})}_{=0;\ (A4)} + \hat{u}' V_{I\theta_2} + V_I \hat{u}'' R's \qquad (29) \\
&\overset{(A3),(A4)}{=} \psi' R' \cdot \underbrace{(\hat{u}' + c_2 \hat{u}'')}_{=0;\ (A2)} - V\psi' R' \hat{u}'' \geq 0.
\end{aligned}
$$

- Next calculate that h_1 is increasing in θ_2 for all DARA-utilities $\hat{u}$:

$$
\frac{\partial h_1}{\partial \theta_2} = R' \frac{\partial S}{\partial \bar{P}} \cdot \frac{\hat{u}''(c_2)}{\hat{u}'(c_2)} + \left(R \frac{\partial S}{\partial \bar{P}} + 1 \right) \cdot \frac{\partial c_2}{\partial \theta_2} \cdot \left(\frac{\hat{u}''(c_2)}{\hat{u}'(c_2)} \right)' > 0 \qquad (30)
$$

  where $\partial c_2 / \partial \theta_2 = R's + V_{\theta_2} > 0$ and $R \cdot \partial S / \partial \bar{P} > -1$ for all θ_1.

- By a similar token, one can show that h_2 is a non-increasing function of θ_2 for all DARA-utilities $\hat{u}$. Moreover, under (A2) to (A4), one can even show that h_2 does not at all depend on θ_2. Namely,

$$
h_2(\theta_1, \theta_2) \overset{(A2)}{=} -\frac{\partial S}{\partial \bar{T}} \cdot \frac{R}{c_2} \overset{(A3),(A4)}{=} -\frac{\partial S}{\partial \bar{T}} \cdot \frac{1}{S + \psi(I)} = \tilde{H}_2(\theta_1),
$$

  with $\tilde{H}_2$ independent of θ_2.

Altogether, this implies by (the standard form of) Chebyshev's Inequality:

$$
\begin{aligned}
\frac{\partial I}{\partial \bar{P}} &= -\frac{1}{M} \cdot \mathbf{E}_{\theta_1} \mathbf{E}_{\theta_2} (g(\theta_1, \theta_2) h_1(\theta_1, \theta_2)) > -\frac{1}{M} \cdot \mathbf{E}_{\theta_1} \big(\underbrace{\mathbf{E}_{\theta_2} g(\theta_1, \theta_2)}_{=:G(\theta_1)} \cdot \underbrace{\mathbf{E}_{\theta_2} h_1(\theta_1, \theta_2)}_{=:H_1(\theta_1)} \big) \\
\frac{\partial I}{\partial \bar{T}} &= -\frac{1}{M} \cdot \mathbf{E}_{\theta_1} \mathbf{E}_{\theta_2} (g(\theta_1, \theta_2) \tilde{H}_2(\theta_1)) = -\frac{1}{M} \cdot \mathbf{E}_{\theta_1} \big(\tilde{H}_2(\theta_1) \cdot \underbrace{\mathbf{E}_{\theta_2} g(\theta_1, \theta_2)}_{=:G(\theta_1)} \big)
\end{aligned}
$$

Now again apply Lemma 1. By the same token as above one shows that $G(\theta_1)$ satisfies (22) and that the functions H_1 and $\tilde{H}_2$ are, respectively, increasing and decreasing in θ_1 (the expectation with respect to θ_2 does not do any harm here). Combine this to obtain

$$
\begin{aligned}
\frac{\partial I}{\partial \bar{P}} &> -M^{-1} \mathbf{E}_{\theta_1} (G \cdot H_1) > -M^{-1} \mathbf{E}_{\theta_1} G \mathbf{E}_{\theta_1} H_1 = 0; \\
\frac{\partial I}{\partial \bar{T}} &= M^{-1} \mathbf{E}_{\theta_1} (G \cdot (-\tilde{H}_2)) < -M^{-1} \mathbf{E}_{\theta_1} G \mathbf{E}_{\theta_1} \tilde{H}_2 = 0.
\end{aligned}
$$

■

Proof of Fact 5

The proof relies on the following

Lemma 2 *Let (A3) be satisfied. Whenever $u(c)$ and $\hat{u}(c)$ exhibit CRRA with respect to c of the same degree and, in particular, under (A1) and (A2), optimal savings $S(I, \bar{T}, \bar{P}, \theta_1)$ satisfy*

$$S(I, \bar{T}, 0, \theta_1) = \sigma(0) \cdot (\pi(I, \theta_1) - \bar{T}) + (\sigma(0) - 1) \cdot \psi(I).$$

where $\sigma : \mathbb{R}_+ \to (0, 1)$ is decreasing in P. Further, $\partial^2 S(I, \bar{T}, 0, \theta_1)/(\partial P \partial x) = 0$ for $x = \bar{T}, \theta_1$.

A **proof** of this lemma is available upon request. The linearity of savings in stochastic settings with CRRA utilities has already been established by Hakansson (1970, Theorem 1). His framework differs from ours, however, by including a safe asset. Hakansson (1970, footnote 5) also notes that for CRRA utility functions *no* analytic solution for the saving function exists if (as in our setting) riskfree assets are unavailable and if the agent earns non-random second-period income (e.g., pensions). Lemma 2 thus only identifies a local feature (at $P = 0$) of the saving function that cannot be generalized. $\diamondsuit$

We continue with the **proof** of Fact 5: Consider a change of the pension formula $(\mathrm{d}\bar{T}, \mathrm{d}\bar{P})$ such that $\mathrm{d}\bar{P} = \kappa \cdot \mathrm{d}\bar{T} > 0$ for some positive constant κ. We are interested in

$$\mathrm{d}I\big|_{\mathrm{d}\bar{P}=\kappa\cdot\mathrm{d}\bar{T}>0} \quad = \quad \mathrm{d}\bar{T} \cdot \left[-\frac{1}{M} \mathbf{E}_{\theta_1} \mathbf{E}_{\theta_2} g(\theta_1, \theta_2) \cdot h_3(\theta_1, \theta_2) \right]$$

where g is defined as in (28a) or (28b). Further,

$$h_3(\theta_1, \theta_2) := \left[\kappa \frac{\partial S}{\partial \bar{P}} + \frac{\partial S}{\partial \bar{T}} + \frac{\kappa}{R} \right] \cdot R \cdot \frac{\hat{u}''(c_2)}{\hat{u}'(c_2)} = -\frac{1}{S + \psi} \left[\kappa \frac{\partial S}{\partial \bar{P}} + \frac{\partial S}{\partial \bar{T}} + \frac{\kappa}{R} \right],$$

where the second expression follows for logarithmic $\hat{u}$. Hence, by integrating,

$$H_3(\theta_1) := \mathbf{E}_{\theta_2} h_3(\theta_1, \theta_2) = -\frac{1}{S + \psi} \left[\kappa \frac{\partial S}{\partial \bar{P}} + \frac{\partial S}{\partial \bar{T}} + \kappa \mathbf{E}_{\theta_2} R^{-1} \right].$$

Next verify that for logarithmic u and with (A3) the function g does not depend on θ_2:

$$g(\theta_1, \theta_2) = \frac{\pi_I + \psi'}{S + \psi} =: G(\theta_1).$$

Hence,

$$\mathbf{E}_{\theta_1} \mathbf{E}_{\theta_2} g(\theta_1, \theta_2) \cdot h_3(\theta_1, \theta_2) = \mathbf{E}_{\theta_1} \left(G(\theta_1) \cdot \mathbf{E}_{\theta_2} h_3(\theta_1, \theta_2) \right) = \mathbf{E}_{\theta_1} G(\theta_1) \cdot H_3(\theta_1).$$

As $\mathbf{E}_{\theta_1} G = 0$ and G is increasing in θ_1, the sign of this expression depends on the monotonicity properties of $H_3(\theta_1)$. Calculate:

$$\begin{aligned} H_3'(\theta_1) &= -\frac{1}{(S+\psi)^2}\left[(s+\psi)\cdot\underbrace{\left(\kappa\frac{\partial^2 S}{\partial\bar{P}\partial\theta_1}+\frac{\partial^2 S}{\partial\bar{T}\partial\theta_1}\right)}_{=0;\,\text{Lemma 2}} - \frac{\partial S}{\partial\theta_1}\cdot\left(\kappa\frac{\partial S}{\partial\bar{P}}+\frac{\partial S}{\partial\bar{T}}+\kappa\mathbf{E}_{\theta_2}R^{-1}\right)\right] \\ &= \frac{\sigma^2(0)\pi_{\theta_1}}{(S+\psi)^2}\cdot\left(\kappa\mathbf{E}_{\theta_2}R^{-1}-1\right), \end{aligned}$$

where we used the properties of the saving function mentioned in Lemma 2. Hence,

$$\mathrm{d}I\big|_{\mathrm{d}\bar{P}=\kappa\cdot\mathrm{d}\bar{T}>0} \gtreqless 0 \quad\Longleftrightarrow\quad \kappa \gtreqless (\mathbf{E}_{\theta_2}R^{-1})^{-1}$$

which completes the proof. ■

Proof of Fact 6

Proof: Implicit differentiation of (19) yields:

$$\frac{\partial I}{\partial\tau} = -\frac{1}{M}\cdot\left\{(A-R)\mathbf{E}_{\theta_1}(\pi_I\hat{u}') + \mathbf{E}_{\theta_1}\left[g(\theta_1)\hat{u}'\cdot\underbrace{\left(R\frac{\hat{u}''}{\hat{u}'}\cdot\left(\frac{\partial S}{\partial\tau}+\frac{A\pi}{R}\right)\right)}_{=:H(\theta_1)}\right]\right\} \tag{31}$$

where

$$g(\theta_1) := \pi_I\cdot((1-\tau)R+\tau A)+V_I.$$

Again, M is a negative cofactor from the SOC for optimal investment. Note that $\mathbf{E}_{\theta_1}(g(\theta_1)\hat{u}') = 0$ from (19). Furthermore, as a function of θ, $g\cdot\hat{u}'$ satisfies the single-crossing property. To apply Chebyshev's Inequality from Lemma 1, we establish the monotonicity properties of $H(\theta_1)$. First verify that for logarithmic utility and under (A3') and (A4), $R\hat{u}''/\hat{u}' = -(s+\psi)^{-1}$. Hence,

$$H'(\theta_1) = -\frac{1}{(s+\psi)^2}\cdot\left[(s+\psi)\cdot\left(\frac{\partial^2 S}{\partial\tau\partial\theta_1}+\frac{A\pi_{\theta_1}}{R}\right)-\frac{\partial S}{\partial\theta_1}\cdot\left(\frac{\partial S}{\partial\tau}+\frac{A\pi}{R}\right)\right]$$

For logarithmic utilities $u(c)=\log(c)$ and $\hat{u}(c)=\delta\log(c)$ savings are given by

$$S = \frac{1}{1+\delta}\left[\pi\cdot((1-\tau)\delta-\tau A/R)-\psi\right].$$

Hence,

$$\begin{aligned} \frac{\partial S}{\partial\theta_1} &= \pi_{\theta_1}\cdot\frac{R\delta(1-\tau)-\tau A}{R(1+\delta)} \\ \frac{\partial S}{\partial\tau}+\frac{A\pi}{R} &= (A-R)\cdot\frac{\delta\pi}{R(1+\delta)} \\ \frac{\partial^2 S}{\partial\theta_1\partial\tau}+\frac{A\pi_{\theta_1}}{R} &= (A-R)\cdot\frac{\delta\pi_{\theta_1}}{R(1+\delta)}, \end{aligned}$$

such that, after some term shuffling, H' simplifies to:

$$H'(\theta_1) = (R - A) \cdot \psi \cdot \pi_{\theta_1} \cdot \frac{\delta^2}{(1+\delta)^2 (S+\psi)^2}.$$

Several cases can be distinguished:

- $\psi \equiv 0$. Then $H' = 0$ or $H(\theta_1) = \bar{H}$ and we get in (31) that

$$\frac{\partial I}{\partial \tau} = -\frac{1}{M} \cdot \Big[(A-R) \cdot \mathbf{E}_{\theta_1}(\pi_I \hat{u}') + \bar{H} \cdot \underbrace{\mathbf{E}_{\theta_1}(g(\theta_1)\hat{u}')}_{=0;\,(19)}\Big] = -\frac{A-R}{M} \cdot \mathbf{E}_{\theta_1}(\pi_I \hat{u}') = 0,$$

  due to the FOC (19) in the case of $V_I = 0$ (for arbitrary values of $(A - R)$).

- $\psi, \psi' > 0$ for all I. Then $\text{sgn} H' = \text{sgn}(R - A)$.

  1. Suppose that $R = A$. Then:

  $$\frac{\partial I}{\partial \tau} = -\frac{A-R}{M} \cdot \mathbf{E}_{\theta_1}(\pi_I \hat{u}') = 0.$$

  2. Suppose that $R < A$. Then:

  $$\begin{aligned} \frac{\partial I}{\partial \tau} &< -\frac{1}{M} \cdot \Big[(A-R)\mathbf{E}_{\theta_1}(\pi_I \hat{u}') + \underbrace{\mathbf{E}_{\theta_1}(g(\theta_1)\hat{u}')}_{=0} \cdot \mathbf{E}_{\theta_1} H(\theta_1)\Big] \\ &= -\frac{A-R}{M} \cdot \mathbf{E}_{\theta_1}(\pi_I \hat{u}') < 0. \end{aligned}$$

     The last inequality follows from the fact that $\mathbf{E}(g\hat{u}') = 0$ implies $\mathbf{E}(\pi_I \hat{u}') < 0$ for $V_I = R\psi' > 0$.

  3. Suppose that $R > A$. Then:

  $$\begin{aligned} \frac{\partial I}{\partial \tau} &> -\frac{1}{M} \cdot \Big[(A-R)\mathbf{E}_{\theta_1}(\pi_I \hat{u}') + \underbrace{\mathbf{E}_{\theta_1}(g(\theta_1)\hat{u}')}_{=0} \cdot \mathbf{E}_{\theta_1} H(\theta_1)\Big] \\ &= -\frac{A-R}{M} \cdot \mathbf{E}_{\theta_1}(\pi_I \hat{u}') > 0, \end{aligned}$$

     by the same argument as before. ■

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