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The Optimal Duration of Contracts

Abstract

We study the optimal duration of contracts in a principal-agent framework with both moral hazard and adverse selection. Agents decide on a contract-specific and non-verifiable investment. Incentive compatibility requires that initial contracts, which serve to screen the ability of newly hired agents, cannot be longer than continuation contracts, offered to successful agents. Initial contracts remain unpaid unless service quality is unobservable to other agents and the share of high-ability agents is high. Optimal durations depend, in non-monotonic ways, on the principal's ow valuation of the agent's service and the share of high-ability agents.

JEL-Codes: J300, L140, J410, D720.

Keywords: contract length, term length, screening.

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1 Introduction

Many contractual relationships consist of a series of contracts which are each of a fixed duration. Furthermore, there is often a peculiar structure in timing and compensation: a low-paid initial contract, which can be of rather short duration, is followed, in case of success, by better-paid renewable contracts, which are usually longer. A firm in need of advertising campaigns may, with a new agency, first go for a short campaign and, if satisfied, order more extensive and expensive campaigns later. In the performing arts, a debuting dancer or actor may initially work for free in a production, in hope of later getting a paid contract; yet, preparing for the role may take a similar effort whether the contract is short or long, paid or unpaid. A curator of exhibitions often needs to engage in complicated negotiations with museums and private collectors around the world; these investments are independent of how long the exhibition is going to run. In these settings, the principal can vary the length and compensation offered after observing performance in the initial contract. In some other settings, most notably in politics, institutional restrictions prohibit this: the term length and compensation of elected politicians is the same whether they are incumbents or challengers. Yet, searching for capable civil servants and advisers, launching the program for the electoral term, or establishing a workable cabinet takes time and effort independently of the term length.

In this paper, we ask what the optimal contract specifications in terms of durations and salaries are from the perspective of a principal who faces both moral hazard and adverse selection problems with her agents but who can replace them after each contract period. We consider a principal who wishes to have a continuous flow of a certain service delivered by an agent. The quality of this service can be low or high but is non-verifiable, implying that the compensation for the agent cannot be conditioned on it. Providing a high-quality service requires the agent to incur a contract-specific effort cost, but the principal cannot observe this investment. There is a large pool of agents who could deliver the service requested by the principal, but only one agent can be hired at a time. Agents may differ in their abilities, which cannot be observed ex ante. An agent's success in providing high-quality service depends on his talent, effort and luck. As even the success of high-ability and industrious agents is stochastic, the principal cannot distinguish whether low-quality service results from incompetence, failure to invest, or bad luck. The principal and an agent sign a contract which specifies a duration and the agent's compensation. After a contract expires, the principal decides whether to offer another contract to the same agent or hire a new agent. We examine three different settings. First, we analyze optimal contracts when service quality is observable (but not verifiable) by other agents. In the second setting the principal and the agent observe service quality, but other potential agents do not; this narrows the scope of incentive-compatible contracts for the principal. Third, we study the setting where the principal cannot vary contract durations and pay and is obliged to offer the same duration and pay in each contract, as in politics.

Contracting parties might have various, not necessarily compatible motives for organizing their relationship with a short initial and a longer continuation contract: agents might be willing to accept a first contract at a pay below its effort costs in expectation that, if successful, they will be offered a longer and better paid contract. From the principal's perspective, asymmetric information on the agent's suitability to deliver the desired service may call for starting contractual relationships with a screening period. Moreover, the principal prefers contracts with limited duration, since agents with tenure tend to slack off. Underpaying agents in initial contracts can help to recoup parts of the information rent that agents capture in subsequent contracts. Alternatively, the principal might view decently-paid follow-up contracts as a price for being able to exploit the agent in initial contracts. These motives shape the design of contracts in different ways. If, for example, an initial contract is just a screening device, then there is no incentive to make it longer than is necessary to find out the agent's type. If an underpaid initial contract is the main source of the principal's rent, the principal would like to have it as long as possible and follow-up contracts as short as possible; the agent's interests are, of course, opposite. Contract durations and remunerations are, thus, linked with each other. Moreover, optimal contract design – the best combination of contract durations and remunerations – is endogenous to the specific situation. It depends on how rare agents with the ability to deliver the desired service are, on the severity of moral hazard problems, and on the value of the service the principal wishes to obtain, relative to the effort cost of the initial investment.

To understand how moral hazard and adverse selection shape the optimal duration of contracts and their remuneration, suppose everybody observes the service quality and the principal's decision whether the current agent is offered a new contract. The agent relies on the principal's reputation for offering a new contract after a successful performance and not offering one after a failure. In this case, the initial contract is always unpaid and the payment in the subsequent contracts is (just) high enough to provide the agent with an incentive to invest in effort. Contract durations depend on the value of the service to the principal and the type-distribution among agents. Specifically, if the share of high-ability agents is low, initial contracts are of the duration minimally needed to find out the agent's type, and continuation contracts are considerably longer. If the share of high-ability agents is high, the principal earns a higher profit from the unpaid initial contract. Reputational concerns, however, prevent the principal from reneging from the implicit promise of a follow-up contract, along the lines of the pioneering contributions by Holmström (1981) and Carmichael (1984).

Optimal contract design differs in the setting where the principal's actions and the service quality she obtains are not observed. The principal needs to pay a salary during the initial screening period to assure agents that a successful performance will be followed by a continuation contract. If positive, then the salary level in the initial contract just makes the principal indifferent between offering a continuation contract to a successful agent and employing a randomly drawn new agent in an initial contract. Finally, if the principal is legally required to only offer contracts of the same duration and pay, as in politics, the optimal contract is shorter and pays a higher wage than the optimal continuation contract in the absence of constraints on durations and payments.

In the previous literature on contracts in multi-period environments, information problems have been limited to either moral hazard (Malcomson and Spinnewyn 1988; Rey and Salanié 1990) or adverse selection (Harris and Holmström 1987). In a discrete-time framework with a series of one-period contracts, Levin (2003) characterizes the optimal relational contract with both moral hazard and adverse selection between a single principal and a single agent. He shows that an optimal sequence of one-period contracts often will employ the same compensation scheme at every date (*stationary contracts*). Calzolari and Spagnolo (2017) extend this to the case of a pool of agents out of which the principal selects a finite subgroup by competitive screening. Contract durations are not discussed, however. To our best knowledge, ours is the first paper that analyzes the optimal duration of contracts in continuous time when moral hazard and adverse selection problems are present simultaneously. We also assume that agents have to decide on a contract-specific investment in each contract spell, as is the case in advertisement campaigns, cultural productions and museum exhibitions.

Our approach differs from much of the literature by assuming that the principal, who only needs one agent, has access to a large pool of potentially suitable agents by whom an under-performing agent might be replaced. Earlier contributions have studied contracting in time between one principal and one agent only (see Guriev and Kvasov 2005; Sannikov 2008, Kvaløy and Olsen 2009; Mason and Välimäki 2015; Garicano and Rayo 2017). Models where agents can be fired – and where probation periods and contract termination play a role – have so far assumed exogenous contract durations (Stiglitz and Weiss 1983; Sadanand et al. 1989; Loh 1994; Spear and Wang 2005). For settings with multiple potential workers of unknown quality, labor economists have since long emphasized the role of initial short-term contracts ("probationary stage") as a mechanism to reach optimal matches and efficient production (Jovanovic 1979; Lazear 1979; Cantor 1988).

In a variety of scenarios, early theoretical approaches to contract length have suggested that contract duration is negatively related to uncertainty, whatever its source (Gray 1978; Canzoneri 1980). As in our framework, these models assume a fixed cost per contract and, thus, provide an incentive for long agreements. Pointing into opposite direction, uncertainty implies that contract-relevant parameters may deviate from their anticipated levels, generating the need for changing the contract. Consequently, unless complete contingent contracts can be written, enforced and re-contracted without cost, contracts are generally of finite duration (Dye 1985). Halonen-Akatwijuka and Hart (2017) consider "continuous" contracts, i.e., contracts of indefinite duration in the sense that they roll over but can be revised at any time at the initiative of either party; in case of revision, the previous contract serves as a reference point.[1]

Contract durations are also important in non-labor contexts. Aghion and Bolton (1987) suggest that incumbent firms can use long-term contracts as a barrier to entry. For patents, understood as contracts between society and innovators temporarily granting an exclusive property right, the optimal duration is also widely discussed (see Denicolò 1996 and Scotchmer 2004, Chapter 4, for surveys). Diamond (1991) argues that short-term contracts

[1]Other approaches challenge the negative nexus between uncertainty and contract duration. In Danziger (1988), risk-neutral firms provide insurance in the form of long-term wage contracts to risk-averse workers, implying that the greater aggregate economic uncertainties are, the longer contracts should run. Harris and Holmström (1987) argue that increased uncertainty may make information gathering and processing more costly and, thus, would call for longer contracts.

might be used by some agents (in particular, borrowers in credit contracts) to signal their high quality by submitting themselves to the risks of renegotiation. Guriev and Klimenko (2015) study the optimal duration of international trade agreements, arguing that spillover effects of the underlying trade-related investments determine whether the agreement should have a fixed duration or be an evergreen contract (i.e., an indefinite contract which can be terminated by either side with notice).

Empirical evidence on the drivers of contract duration is scarce. Wallace (2001) shows that the relationship between uncertainty and contract length is inconclusive. Joskow (1987), Crocker and Masten (1988), Brickley et al. (2006), and Lin and Yang (2016), which study, respectively, the contract duration for coal, gas, franchises and baseball players, find that contract length is determined by the rents from the contract, the incentive concerns for non-observable effort and the need for flexibility. These findings are corroborated by Bandiera (2007), which seems to be the first empirical study that analyzes both contract duration and the compensation scheme (for land tenancy contracts in Italy); the latter is, however, only captured by the distinction between fixed-rent and share-cropping contracts. For professional football players, Tang (2013) finds that contract length is longer the higher is the expected rent from a player for the team owner; the study also reports that first contracts for a player are typically shorter and less well-paid than second ones.

These empirical observations are accommodated by our theoretical model. However, our analysis suggests that many of the observed monotonous relationships need not hold in general. For example, that contract duration is longer the more valuable the project is for the principal is only true *conditional* on having found a high-ability agent, i.e., only holds for (unconstrained) continuation contracts. For initial contracts, it does not hold, and if the durations of initial and continuation contracts are interlinked, the monotonicity need not even hold for continuation contracts (see Section 3.3 for details). In addition, our model also emphasizes that the ability distribution in the pool of agents shapes contract design; this prediction remains (to our knowledge) still to be empirically checked (see Section 3.4).

The rest of the paper is structured as follows: Section 2 sets up our model with the principal having reputation concerns, and Section 3 presents optimal contract structures and their comparative statics for this scenario. Section 4 analyzes optimal contracts without reputation concerns. Section 5 derives optimal contracts when all contracts have to be of equal lengths. Section 6 concludes. Proofs are collected in appendices.

2 Set-up

Primitives. We consider a model in continuous time with one principal (referred to as "she") and a large number of potential agents (referred to as "he"). Both the principal and the agents are risk-neutral, live infinitely long, and have the same rate of time preference $\delta \geq 0$. At any instant in time, the principal wishes to have a certain service provided; for this, she can employ precisely one agent at a time. Agents come in two types, referred to as high and low ability. Only high-ability agents have the potential to deliver high-quality service; low-ability agents never succeed. There is potential adverse selection: at the beginning of the contractual relation, the agent's type is his private information, and the principal can infer the agent's type with certainty only after good performance. It is common knowledge that the proportion of high-ability agents is $\mu \in (0, 1)$.[2]

The relationship between principal and an agent is governed by (a sequence of) contracts that each specify duration and compensation. A key assumption is that an agent has to make a contract-specific investment at the beginning of each new contract. In the various applications of our model, such costs arise, e.g., from putting together a new team, drafting a research proposal in academia, setting up a policy agenda for the electoral term (and in the American system, for getting the top civil servants confirmed), designing a new campaign in advertising, choreographing a new repertoire in the performing arts, developing a new strategy in consulting, etc.[3] The service which the agent is supposed to deliver to the principal can be of either high or low quality. To the principal, the instantaneous value of the service is zero if quality is low and $v > 0$ if the quality is high. Agents decide whether to exert effort at the beginning of a contract. Exerting effort involves non-monetary cost $c > 0$ to the agent. The disutility of work without exerting effort is normalized to zero; we could allow for an additional constant flow

[2] With $\mu = 1$ (all agents have high ability), optimal contracts only have to cope with moral hazard; screening issues do not arise. In this case, contract duration will be optimally set such as to minimize the instantaneous wage cost (see (1) below).

[3] The assumption of an exogenous and irreversible cost for the agent in each contract spell makes repetitive contracts expensive; starting with Crocker and Masten (1988), it can be found in several studies on the relationship between contract duration and uncertainty. Fixed bargaining or recontracting costs – as in Dye (1985) – formally have a similar effect.

disutility from work for the agent without affecting our results. For a low-ability agent, effort is futile: he never succeeds to deliver high-quality service. As a consequence, a low-ability agent optimally never exerts any effort. For a high-ability agent, delivering a high-quality service requires effort. Output quality is stochastic, however. Even with effort, a high quality is only obtained with a certain likelihood $p \in (0, 1)$.

Contracts. Any contract offered by the principal specifies its duration $t \geq 0$ and the instantaneous compensation $w \geq 0$ for the agent during this duration. Agents have no wealth and cannot be punished so that w must be non-negative at each instant. We assume that payments from the principal to an agent and contract durations are verifiable by third parties. Service quality is observable to both the principal and the agent but not verifiable. Effort can neither be observed nor verified; note, however, that the production technology implies that a high service quality reveals that effort has been spent and that the agent is of high ability.

After the current contract expires, the principal decides whether to offer a new contract to the same agent. We assume that she must honor the contract irrespective of the realized service quality, and rule out dismissal or renegotiation of the contract before it expires. In the field of politics, this is in line with fixed electoral terms. Otherwise the principal would like to replace a failed agent immediately upon observing a low service quality and render a contract permanent upon observing a high quality.

Each contracting episode proceeds as follows: first, the principal offers a contract, stipulating duration and compensation to an agent who may be new or who may have served the principal for previous contract periods with an impeccable track record. Second, high-ability agents under contract choose whether to exert effort; low-ability agents invariably shirk. Service quality is realized. An agent who fails to deliver high-quality output (i.e., low-ability agents, high-ability agents who shirked, and high-ability agents who exerted effort but had bad luck) are dismissed and replaced by a newly hired agent, randomly drawn from the pool of agents (as the pool is large, we assume that dismissed agents have zero probability of being drawn again). Successful agents will be offered a new contract ("continuation contract") which may differ from their initial, "screening" contract as their success has revealed them to be of high ability.

In Sections 2 and 3 we assume that all agents observe the service quality and whether successful agents are indeed offered a new contract while unsuccessful agents are not. If the principal did not offer a new contract after a successful initial contract, future agents would recall this and no longer invest in effort and the principal would not be able to continue with her projects. For contract design, this means that offering a contract to a successful agent must be preferable for the principal to aborting contractual relations entirely (see Eq. (7) below). In Section 4 we change the scenario and assume that service quality is not observable by third parties. For contract design, this implies that offering a contract to a successful agent must be preferable for the principal to re-starting the contractual relation with a fresh agent of unknown quality.

We denote the durations of initial and continuation contract by s and t, respectively. The following notation will prove convenient: when x denotes a duration, we write $X := 1 - \mathrm{e}^{-\delta x}$. This function is strictly monotonic and maps durations $x \in [0, \infty)$ to the unit interval: $X \in [0, 1)$. It allows us to work with "discounted" lengths of a contract with real-time duration x by $\int_0^x \mathrm{e}^{-\delta\tau} \mathrm{d}\tau = X/\delta$. For simplicity, we shall henceforth discuss contract durations in terms of capitalized, discounted time spans (rather than in terms of real time spans).

We assume that there is an exogenous minimum duration of contracts: $S, T \geq S_{min}$ for some $S_{min} > 0$. We shall also assume that the minimum duration is not too long. Specifically, $S_{min} < 1/2$.

The agent. Suppose an agent has been offered a continuation contract of duration T with wage w. The value of that contract when the agent shirks (and, thus, will for sure not be offered any subsequent contract) is:

$$\int_0^t w\mathrm{e}^{-\delta\tau} \mathrm{d}\tau = \frac{wT}{\delta}.$$

When exerting effort, the agent will successfully complete the project with probability p and then be offered a continuation contract with identical conditions (T, w) as before. In that case the value of the contract for the agent is

$$\pi = \int_0^t w\mathrm{e}^{-\delta\tau} \mathrm{d}\tau - c + p\mathrm{e}^{-\delta t}\pi.$$

Using $T = 1 - e^{-\delta t}$ and rearranging leads to

$$\pi = \frac{wT - \delta c}{\delta\left(1 - p(1 - T)\right)}.$$

Comparing the payoffs from shirking and non-shirking, an agent will not shirk whenever

$$w \geq w(T) := \frac{\delta c}{pT(1-T)}. \tag{1}$$

For further use, note that the required wage rate in the continuation contract is minimized when $T = 1/2$. As the principal has no incentive to pay wages higher than $w(T)$, the agent's valuation of a renewable continuation contract is given by

$$\pi|_{w=w(T)} = \frac{\delta c T}{\delta p T(1-T)} = \frac{c}{p(1-T)}.$$

The initial contract of duration S pays wage $w^s \geq 0$. The agent will only exert effort in the initial contract if the effort costs do not exceed the expected present value of a continuation contract, i.e., if

$$c \leq p(1-S) \cdot \frac{c}{p(1-T)}.$$

This simplifies to

$$S \leq T, \tag{2}$$

i.e., if the wage is given by (1), the initial contract may not be longer than the continuation contract.

The principal. For the principal, the values V of the continuation contract and Z of the initial contract are given by:

$$V = \frac{1}{\delta}(pv - w)T + p(1-T)V + (1-p)(1-T)Z, \tag{3}$$

$$Z = \frac{1}{\delta}(\mu pv - w^s)S + \mu p(1-S)V + (1-\mu p)(1-S)Z. \tag{4}$$

In (3), the first term on the right-hand side is the expected present value (at the beginning of the continuation contract) of the current contract. The second term captures a renewal of this contract, provided that the agent (who is known to be of high ability) succeeds in delivering high-quality service. For the case of an unsuccessful agent, the third item captures that an initial contract will be offered to a newly hired agent. In (4), the first term on the right-hand side is the expected present value (as of now) of the current initial contract. The second and third terms capture that, upon expiry after duration S, a continuation contract will be offered to an agent who is successful while, otherwise, a re-start with a new agent will take place.

Solving (3) and (4) for V and Z, using $w(T)$ from (1), gives:

$$\begin{aligned} Z(S,T) &= \frac{1}{\delta D(S,T)} \cdot \Big[-w^s(1-T)(1-p(1-T))S - c\delta\mu(1-S) \\ &\quad + v\mu p(1-T)(pT+(1-p)S)\Big] \qquad (5) \\ V(S,T) &= \frac{1}{p\delta D(S,T)} \cdot \Big[-w^s p(1-T)^2(1-p)S - c\delta(S+\mu p(1-S)) \\ &\quad + vp^2(1-T)\left(ST+\mu(p(T-S)+S(1-T))\right)\Big]. \qquad (6) \end{aligned}$$

Here, the denominator

$$D(S,T) := (1-T)(S(1-p(1-T)) + \mu p(1-S)T)$$

is positive for all $(S,T) \in (0,1)^2$. To ensure that the principal will indeed offer a continuation contract after a successful initial contract, it must be true that the principal's value from the continuation contract is at least as large as aborting contractual relations entirely, i.e.,

$$V(S,T) \geq 0. \qquad (7)$$

We will henceforth refer to (7) as the principal's incentive compatibility (IC) constraint. Economically underlying (7) is the assumption that the service quality which agents deliver is observable by third parties; discontinuing the contractual relationship with a successful agent would then be an obvious breach of the principal's promise, inherent in (3), to offer a follow-up contract as long as the agent remains successful. Such loss in credibility would jeopardize the principal's future hiring plans.

When service quality is *not* observable by outsiders (but only to the principal and the current agent), a continuation contract not only needs to pay off for the principal ($V(S,T) \geq 0$) but must offer her at least as high payoffs as a re-start of the contractual relation with an agent of unknown quality, i.e., $V(S,T) \geq Z(S,T)$. We will discuss this scenario in Section 4.

3 Optimal contract regimes with observable service quality

3.1 The optimization problem

We are interested in durations of initial and continuation contracts that maximize the principal's payoff and are incentive-compatible for all parties. As both Z and V are strictly decreasing in w^s, a positive wage in the initial contract both reduces the principal's payoff and sharpens her IC constraint (7). Hence, initial contracts optimally remain unpaid:

$$w^s = 0.$$

Optimal contracts, thus, involve deferred compensation, as in Lazear (1979): (high-ability) agents are made to exert effort during the unpaid initial contract by the prospect of paid continuation contract(s) from which they can earn an informational rent.

Searching for optimal contract design boils down to maximizing, by choice of (S, T),

$$Z(S,T) = \frac{1}{\delta D(S,T)} \left[-c\delta\mu(1-S) + v\mu p(1-T)(pT + (1-p)S) \right], \tag{8}$$

subject to

- the IC constraints (2) and (7), where $w^s = 0$ in $V(S,T)$;
- the principal's participation constraint
$$Z(S,T) \geq 0; \tag{9}$$
- and the minimum duration constraint $S \geq S_{min}$.

We denote optimal durations by (S^*, T^*). The corresponding wage in the continuation contract is then given by $w(T^*)$ from (1).

The optimal structures and durations of (unpaid) initial and (paid) continuation contracts vary with the model parameters (v, c, p, μ, δ). It is helpful to organize parameters by means of

$$\Gamma := (1-\mu)p^2 v - 4c\delta.$$

Obviously, Γ is larger the larger v or p are and the smaller c, δ, or μ are. Given the monotonicity patterns of Γ in the instantaneous value, v, of the project and the project costs, c, we will henceforth call projects with $\Gamma > 0$ "highly profitable" and such with $\Gamma < 0$ "moderately profitable".[4] The knife-edge case $\Gamma = 0$ will mark an important threshold for contract design.

3.2 Optimal durations and structures

An optimal contract design balances the costs and benefits of longer contracts. If the service value is sufficiently high or the share of high-ability agents sufficiently low (captured by $\Gamma > 0$), the main purpose of initial contracts is to detect high-quality agent. Consequently, initial contracts are as short as possible. Continuation contracts, instead, are long. Their duration is limited, however, as a very long contract would require a very high wage to incentivize the agent. If, by contrast, the service value is lower or high-ability agents are relatively abundant (i.e., when $\Gamma < 0$), the principal mainly benefits from unpaid initial contracts. According to the agent's IC constraint, initial contracts must not be longer than an eventual continuation contract. Proposition 1 formally describes these general properties.

Proposition 1 *Suppose that the principal's participation constraint (9) is not binding at the optimal contract.*

1. *For highly profitable projects ($\Gamma > 0$), initial contracts are optimally as short as possible: $S^* = S_{min}$. Optimal continuation contracts have a duration longer than* $1/2$.

2. *If $\Gamma = 0$, all durations $S^* \in [S_{min}, 1/2)$ are equally optimal for the initial contract, and a continuation contract has optimal duration $T^* = 1/2$.*

3. *For moderately profitable projects ($\Gamma < 0$), initial and continuation contracts have equal length $T^* = S^*$. This duration can be longer or shorter than $T = 1/2$, depending on parameters.*

At the knife-edge case $\Gamma = 0$, the principal earns, at an optimally designed contract, the same expected profit flow from an initial contract with zero wage

[4] Non-profitable projects are such that the principal's participation constraint cannot be satisfied.

as from a continuation contract with a high-ability agent who receives positive wage (see Eq. (15)). Hence, the optimal continuation contract minimizes the wage costs $w(T)$. The principal's payoff can then be understood as the present expected value of the project in a hypothetical case where she continuously employs, for a zero wage, a randomly chosen agent who supplies effort: $Z = pv/\delta$ (where v is connected to the other parameters via the condition $\Gamma = 0$).

For highly profitable projects ($\Gamma > 0$), the principal earns higher expected profits from a salaried high-quality agent than from an unpaid agent of unknown quality. The initial contract only serves to screen for high-quality agents, which should happen as quickly as possible ($S^* = S_{min}$).

For moderately profitable projects ($\Gamma < 0$), the principal earns a higher expected profit from the unpaid initial contract than from the continuation contract. Hence, after each contract spell she would prefer a new random draw from the pool of agents. Her only reason for offering a paid continuation contract is that otherwise agents would not exert effort in the unpaid initial contract. As the IC constraint (2) rules out that the initial contract is longer than the continuation contract, initial and continuation contracts have equal durations.

The proviso in Proposition 1 that contracts are at all provided is necessary. The proof of item 2 reveals that the principal's participation constraint (9) is satisfied when $\Gamma = 0$ and, thus, for $\Gamma > 0$ as well. For negative values of Γ it is, however, no longer clear whether it will be feasible for the principal to offer any contract at all that earns her a non-negative payoff and attracts agents (see Proposition 3).

Proposition 1 conveys that optimal contract structures qualitatively change when Γ switches its sign. For positive Γ, the initial contract mainly serves screening purposes; the more substantial benefit to the principal arises from the continuation contracts. For negative Γ, the benefit to the principal mainly comes from the unpaid initial contract. For principals who are unsure whether their projects are of high or not-so-high value ($\Gamma > 0$ or $\Gamma < 0$), this discontinuity may pose the problem of which type of contract to offer to their agents.

3.3 The effect of the project value

So far only little is known about how contract durations vary across different projects, i.e., when project parameters, collected in Γ, vary. In this section, we focus on the role of v, the principal's flow value of a successful project for contract design. Obviously,

$$\Gamma \geq 0 \qquad \Longleftrightarrow \qquad v \geq \hat{v} := \frac{4c\delta}{(1-\mu)p^2}.$$

Hence, *given* the values of the other parameters, $\hat{v}$ marks the threshold between moderately and highly profitable projects at which contract regimes undergo their structural change. Figure 1 visualizes the results to come.

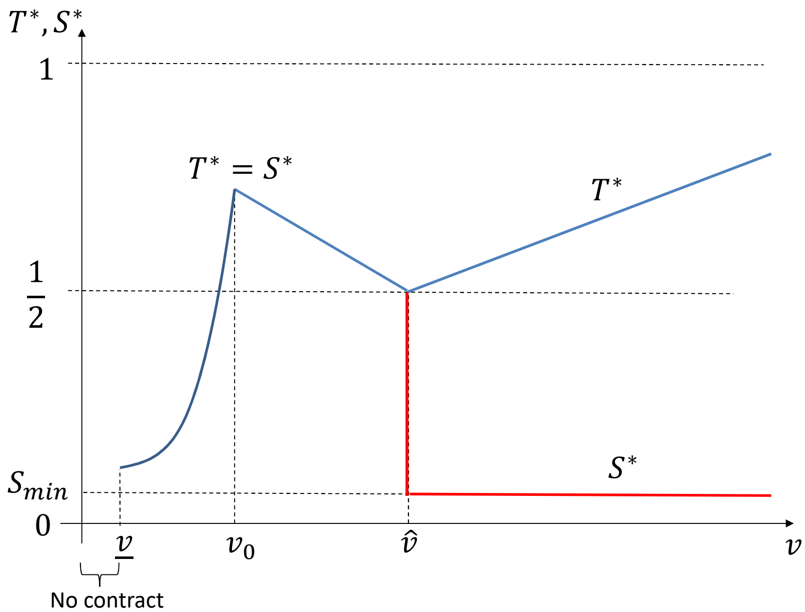

Figure 1: Optimal contract durations

For highly profitable projects ($v > \hat{v}$), initial contracts should be as short as possible and continuation contracts have a duration longer than 1/2 (first item of Proposition 1). Since the main source of the principal's benefit is the continued employment of agents who have been found to be of high ability, more valuable projects call for longer continuation contracts. As the wage

$w(T)$ increases with T for $T > 1/2$ (see (1)), these contracts will also be better paid.

Proposition 2 *For highly profitable projects ($v > \hat{v}$), the optimal duration of continuation contracts gets longer and the pay gets higher if the instantaneous project value, v, rises.*

For lower-value projects ($v < \hat{v}$), contract design is more complex as here, the various IC and participation constraints are binding. Generally, from the third item of Proposition 1, initial and continuation contracts have equal length and the principal's payoff will be mainly earned from the (unpaid) initial contract. In fact, it is not guaranteed that the continuation contract adds positively to the principal's payoffs at all, implying that her IC constraint (7) may bind. Finally, with low-value projects the principal's participation constraint – total payoffs must not be negative – has to be observed.

These considerations shape optimal contract design as follows: for projects with a very low flow value (in Figure 1: for $v < \underline{v}$), no contractual relationship will be established as there is no feasible duration such that the principal earns non-negative payoffs and agents are willing to accept the contract.[5]

For projects with somewhat higher flow value (in Figure 1: if $\underline{v} \leq v \leq v_0$), contracts will be offered but the principal will not make money from the continuation contract and earns her entire payoff from the unpaid initial contract.[6] Essentially, the principal would like to have the initial contract for as long as possible and the continuation contract for as short as possible. The agent's IC constraint $S \geq T$ forbids such a structure, however, and forces the principal to offer equal-length initial and continuation contracts. Their duration is the longest one that is compatible with non-negative continuation payoffs V. For low-value projects, this is quite short. In particular, for the smallest v such that contracts are offered, we have $T^* < 1/2$. When v increases, contracts will get longer and, eventually, will have duration longer than 1/2.

For projects with higher flow value (in Figure 1: if $v \geq v_0$), the principal will earn money both from the continuation and the initial contract. The

[5]Formally, $\underline{v}$ is the smallest flow value v such that the principal will earn non-negative profits at an optimal contract; see (29) in Appendix A.3.

[6]Formally, v_0 is the smallest flow value v such that the principal's payoff, V, from an optimally designed continuation contract is non-negative; see (27) in Appendix A.3.

constraint $V \geq 0$ stops to bite. Contracts optimally get shorter again when v gets larger. The intuition is that both the initial contract with a high-performing agent and the continuation contract are profitable, reducing the motivation to push for long initial contracts. Moreover, shortening the (continuation) contract lowers the wage (recall that $w(T)$ increases in T when $T > 1/2$).

Proposition 3 summarizes the qualitative properties of optimal contract durations for moderately profitable projects in terms of v as well as the parameter range in which there is no contracting.

Proposition 3 *There exist values $0 < \underline{v} < v_0 < \hat{v}$ such that:*

1. *If $v < \underline{v}$, no contract will be offered.*
2. *If $\underline{v} \leq v \leq v_0$, the principal only earns money from the initial contract. The (identical) duration of initial and continuation contract increases in v. It is shorter than $1/2$ at $v = \underline{v}$ but larger than $1/2$ at $v = v_0$.*
3. *If $v_0 < v < \hat{v}$, the principal earns money from both initial and continuation contract. The (identical) duration of initial and continuation contract is longer than $1/2$. Contracts optimally are shorter the higher is v.*

Precise definitions of $\underline{v}$ and v_0 and of optimal contract durations are provided in the proof of Proposition 3. They allow for both numerical calculations and for comparative statics with respect to the other parameters of our model.

While Propositions 2 and 3 are – as our model itself – presented in terms of the principal hiring only one agent, they have interesting implications also when the principal needs to fill a certain number of positions. A question is then how the staff is divided between new workers in initial contracts and longer-serving workers in continuation contracts. Such a situation could arise in consulting firms, but also in arts (think of an opera house or a theater choosing directors and conductors and other professionals in charge of different parts of the repertoire, a film studio choosing directors for various movies or a museum hiring curators to put together big exhibitions). There are also many high-ranking public service positions that are filled with fixed-term contracts that can, but need not be renewed. From Propositions 2 and 3, the value of the service flow affects both the team composition and the duration of the initial and continuation contracts. If this value is relatively low ($v < \hat{v}$), the principal mainly or only gains from initial contracts and would, thus, like to see a high turnover. This might be the case in consulting firms, where the firm makes profit from new employees and is glad to see them leave after some time. The principal is restricted in this by having to offer continuation contracts to successful agents to keep up her reputation and to be able to attract new high-ability agents. With a higher value of v ($v > \hat{v}$), the principal gains more from agents in continuation contracts. New entrants are offered contracts with minimum length needed to find out their ability. Those performing well are offered continuation contracts.

As a consequence, personnel turnover and structures differ considerably between principals with high-value projects ($v > \hat{v}$) and with low-value projects. The former will see quite some fluctuations since their junior staff is hired only for the shortest time it takes to detect whether they are successful; only a fraction of μp of all junior staff will be offered a longer contract. By contrast, for $v < \hat{v}$, junior and senior staff both stay longer than S_{min}. Firms make no distinction in contract duration between junior and longer-serving staff. Seniority is only visible in pay; juniors have to work unpaid longer than the minimum duration that is necessary to find out whether they are of high ability.

3.4 The effect of the ability distribution

Proposition 1 also allows to discuss contract design for different shares of high-ability agents, generating a number of testable predictions. Suppose, first, that high-quality agents are rare: μ is small (in the sense that, given the other project parameters, $\Gamma > 0$).[7] Then it is optimal for the principal to have initial contracts that are as short as possible in order to be able to get rid of the frequent non-performing agents rapidly. In the seldom case a high-ability agent has been detected, he will be offered a rather long, paid continuation contract. If, by contrast, high-quality agents are relatively frequent (μ is large), screening is a minor issue; the principal's benefit from the initial contract comes from its being unpaid. The agent's IC constraint limits the extent to which the principal can exploit this benefit.

For continuation contracts, optimal duration varies non-monotonically in

[7] Observe that, depending on the other parameters, this case need not arise: if p^2v is small enough, relative to $c\delta$, Γ might not become positive even if μ approaches zero.

μ. If high-ability agents are rare (μ is small), a principal who was just lucky enough to find a high-quality agent would like to keep him for rather long. This effect is stronger the smaller μ is: the rarer a (rare) high-ability agent, the longer his contract. Now assume that μ is already large, making capable agents relatively easy to find. As the duration of continuation contracts follows the duration of initial contracts, they get longer if μ increases, simply because the principal's expected gain from the unpaid contract rises. Longer initial and continuation contracts allow the principal to compensate agents for their investment costs c less frequently.

Changes in μ can also be interpreted in terms of technological change. For example, we could interpret moving from low to high values of μ as a professionalization of the trade (think of marketing agencies becoming more professional), making available a larger share of high-quality agencies. At some point, this means a switch from initial contracts of a short length to initial and follow-up contracts being of the same length.

An opposite phenomenon comes, for example, from the proliferation of technologies that allow for a fast and widespread distribution of songs, movies, and broadcasts of live performances. This means that those at the top of the distribution capture an even larger audience, at the cost of those who previously served local markets. Rosen (1981) points out that the gap between what the superstars at the top and everyone else earns has increased dramatically. MacDonald (1988) suggests that if there is substantial uncertainty about individual performance and past performance is correlated with future outcomes, only the young enter the occupation and only the successful stay on. This generates a similar compensation structure as our model: young entrants earn low wages (in our model, nothing during the initial contract), while those who are successful are rewarded highly. Technological innovations also reduced entry barriers in many areas: people themselves can now publish their videos, songs, or books via Youtube or Amazon. The number of people who try to start an artistic career is nowadays much higher than in the past (μ decreased). This means that initial contracts become shorter, as evidenced by the increasing number of short-lived starlets who, having more-or-less unsuccessfully produced a single song or book, fall into oblivion. Those at the top in an artistic profession, on the other hand, earn much more than in the past.

4 Optimal contracts when service quality is not observable to third parties

The principal's IC constraint for the continuation contract so far was given by $V(S,T) \geq 0$: offering a continuation contract must not harm the principal. This constraint conveyed that the principal would – in order not to run into difficulties in recruiting new agents in the future – not break her promise and opportunistically discontinue the contractual relationship with a successful agent. The underlying reason is that service quality is observable by third parties.

Without such observability, a continuation contract – which would only be offered to (successful) agents of high quality – not only needs to be worthwhile as such ($V(S,T) \geq 0$) but also must offer at least as high payoffs to the principal as a re-start of the contractual relation with an agent of unknown quality. I.e., with unobservable service quality the principal's IC constraint (7) sharpens to

$$V(S,T) \geq Z(S,T). \tag{10}$$

By (3) and (4) this is equivalent to:

$$w^s + pv(1-\mu) \geq w.$$

The principal chooses (S,T) to maximize (4), obeying $w^s \geq 0$, conditions (1) and (10), and the participation constraint $Z \geq 0$. The following result characterizes optimal contract design in terms of v:

Proposition 4 *Suppose that service quality is not observable to third parties.*

1. *If $\Gamma < -4\mu\delta c$, no contracts will be offered.*

2. *If $-4\mu\delta c \leq \Gamma \leq 0$, all durations $S \in [S_{min}, 1/2)$ are equally optimal for the initial contract. The optimal continuation contract has length $T^* = 1/2$.*

   Wages in the continuation and the initial contract are given by $w(1/2) = 4\delta c/p$ and $w^s = 4\delta c/p - pv(1-\mu)$.

3. *If $\Gamma > 0$, contract durations are as in Proposition 1. In particular, $S^* = S_{min}$ and $T^* > 1/2$.*

   Initial contracts remain unpaid ($w^s = 0$), wages paid in the continuation contract increase with v.

In terms of v, Proposition 4 is illustrated in Figure 2.

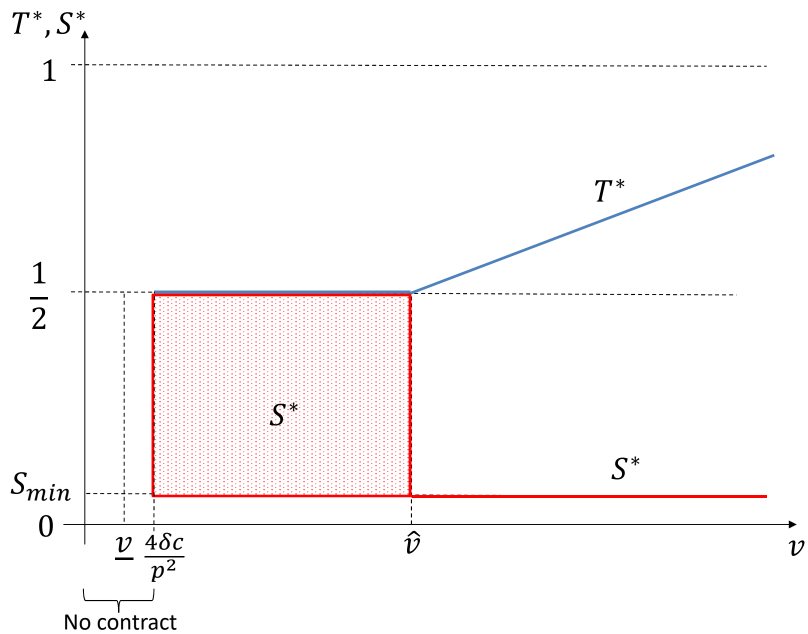

Figure 2: Optimal contract durations with unobservable quality

For high-value projects ($\Gamma > 0$), the non-observability of service quality does not affect optimal contract structures: item 1 in Proposition 1 and item 3 in Proposition 4 do not differ. For low-value projects ($\Gamma \leq 0$), where, with observability, the principal reaps the main benefit from unpaid initial contracts, incentive compatibility for the principal now requires that the initial contract pays a wage, although a lower one than the continuation contract (item 2 in Proposition 4). Given the indifference with respect to S, one particularly simple optimal contract design is to have all contracts last for duration 1/2, but to differentiate pay between newly hired and experienced agents.

5 Contracts in politics: equal pay and equal durations

So far, we have analyzed the case in which initial and continuation contracts are allowed to have different lengths and to pay different wages to the agent. In many relevant applications – such as politics –, such flexibility does not prevail. Rather, initial and continuation contracts have to offer equal conditions, both in terms of duration and pay, whether an incumbent or a challenger is elected. Previous literature on the effects of term lengths in politics has focused on the moral hazard problem and viewed an election as a referendum on the incumbent's performance; see the pioneering contributions by Key (1966), Barro (1973) and Ferejohn (1986), and the more recent analyses by Besley (2006) and Dal Bó and Rossi (2011). Our paper adds adverse selection issues.[8]

Compared to the previous scenario, contract design now faces two additional constraints: $w^s = w$ and $S = T$. They each affect contract design in different ways, but in combination their impact is unambiguous:

Proposition 5 *Suppose that $S = T$ and $w^s = w$ must hold. Then optimal contract duration satisfies $S^* = T^* < 1/2$ for all v and decreases when the flow value v rises.*

Proposition 5 is sketched in Figure 3.

Proposition 5 conveys that for high-value projects, continuation contracts are shorter than in the unconstrained case (cf. the first item of Proposition 1). The reason is that now the only way for the principal to get rid of low-ability agents as quickly as possible after the screening is to shorten contract duration for everyone.[9] The optimal term length balances the marginal benefit from a shorter contract in terms of faster screening of first-time candidates and the marginal cost of having to compensate agents for investment costs

[8] There are a few earlier steps in this direction, with two-period models. Rogoff and Sibert (1988) and Rogoff (1990) introduce models in which politicians signal their competence through policy choices when the electoral term lasts two periods. We add to this literature by endogenizing the term structure and analyzing an infinite sequence of contracts.

[9] Our analysis is, thus, not applicable to jurisdictions (like, e.g., California) where citizens can recall a politician through a referendum or countries that follow the British tradition and give some leeway to the government in when to call the next election. However, such provisions are rare.

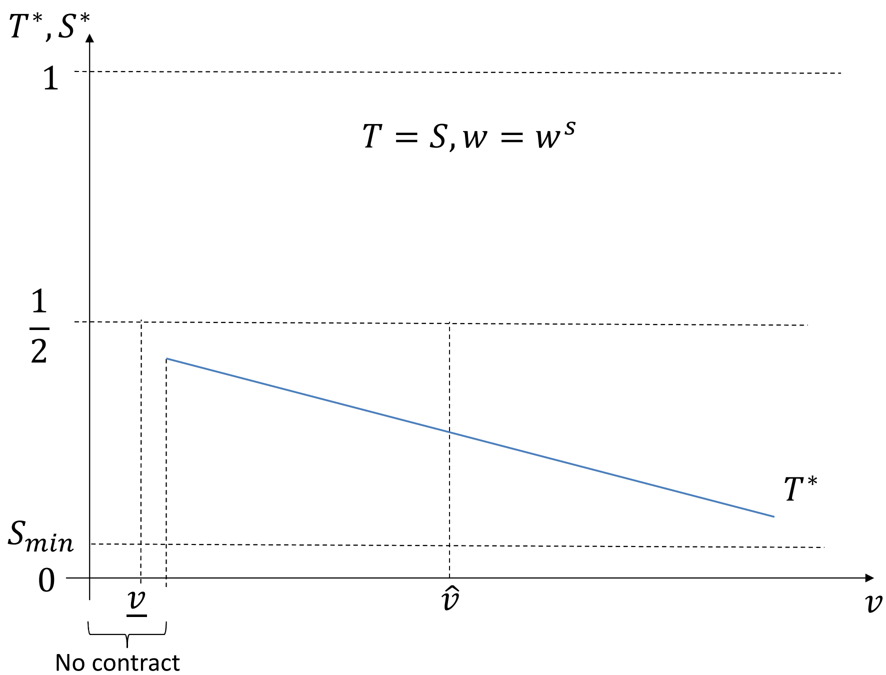

Figure 3: Optimal contracts with equal pay and duration

more often, as well as having a shorter expected total duration of continuation contracts after a high-quality service during the initial contract.

Why is term length restricted to be the same for incumbents and challengers in politics? One possible explanation is that having a different term length in legislatures would require carrying out elections at different times, and this in turn could be expected to reduce turnout (note that the turnout in mid-term elections in the United States is already much lower than in presidential election years). In addition, longer term length in continuation contracts could further entrench incumbents. Finally, voters have different views about desirable policies, which makes coordination more difficult.

Informal institutions in politics may allow different compensations to a certain extent, even when term length would remain the same. Seniority plays an important role in the allocation of powerful committee chairman positions in the United States Congress, but it is also of importance in many other countries. This way, effective compensation in continuation contracts may exceed that in initial contracts. Our results show that it can be in the principal's interest to pay higher compensation after re-election than during the first term. However, this does not imply that it would be efficient to pay such a compensation through ideological rents that committee chairmanship conveys, rather than monetary payments. Alesina and Spear (1988) suggest that political parties can introduce transfer schemes in which younger politicians of the same party provide incentives to the office holder who cannot be re-elected by transfers that are conditional on chosen policies. Their focus is on moral hazard problem, with the policy-maker elected for one period only. Gersbach (2004) shows that allowing politicians to offer incentive contracts that become effective upon re-election helps to alleviate the incentive conflict between voters and politicians. Finally, part of the compensation of elected politicians may arise from job opportunities after leaving politics. If such opportunities are better after delivering high-quality performance, they incentivize politicians to invest effort.

6 Discussion and conclusion

Many long-term relationships are organized into sequences of shorter-duration contracts. We provide a simple incomplete contracting explanation for this design and analyze the optimal duration and compensation structure. Unlike earlier contributions, we allow for both moral hazard and adverse selection problems to be present, and for the principal to change the agent after each contract.

Our model predicts that when all potential agents can observe realized service quality and there are no constraints on contract design, the initial contract is always unpaid. An example could be a debuting actor or dancer working first for free in a production, hoping to get a paid contract later. If high-ability agents are rare and the value of the service is high, the initial contract is just of the minimum duration needed to find out whether the agent is of high ability. If high-ability agents are more common, initial contracts have the same length as continuation contracts. Here, the principal makes a bigger profit from the initial contract than from the continuation contracts, and offers continuation contracts just to maintain her reputation. Requiring that initial and continuation contracts have to be of equal length and pay, as in politics, shortens the optimal duration of continuation contracts, and increases pay above what would prevail if continuation contracts were allowed to differ from initial contracts.

If other agents cannot observe service quality and whether a well-performing agent receives a new offer, an additional constraint becomes relevant: the principal must not make higher expected profit from the initial contract than from continuation contracts. If the share of high-ability agents is sufficiently high, the principal may need to make a payment even during the initial contract to ensure agents that a successful performance is rewarded with a new contract.

With a suitable re-interpretation, our model can be applied to several settings. Consider research funding: society wants to receive high-quality research, but even high-ability researchers can be unlucky or shirk. It is unknown ex ante who are the good researchers, and even if a funding body can evaluate whether realized research has been of high quality, quality cannot be verified in court and payments cannot be conditional on quality. The funding body can require that young researchers have to first prove themselves, during an initial contract with no or low pay. Depending on the field, this can correspond to doctoral studies, or doctoral studies plus a post-doc. Now interpret the contract-specific cost as writing a funding proposal for the following period, writing a compelling final report or any other steps to convince the funding body to continue its funding. A successful person then gets a subsequent paid contract. During this contract, he or she again has to do research and write a new proposal, should he or she wish to obtain a new round of funding, which will only be given to the successful ones. The funding body faces a tradeoff: shorter funding periods require scientists to invest in fixed application and reporting costs more often, but also allow getting rid of unsuccessful agents sooner. While our basic model predicts no pay during the initial contract, the presence of borrowing constraints may mean that also the initial contract has to be paid, although less than subsequent contracts. This, in turn, may encourage also some potential researchers of low abilities to enter the initial contract, even if they subsequently drop out of academia.

A central assumption in our model is that contracts cannot be renegotiated after service quality is observed. To take the example of advertising campaigns, it could be that the principal has to pay for the ads beforehand, and it is not possible to plan a new campaign sufficiently fast even if observing that the current one is unsuccessful. Similarly, if an individual actor or dancer turns out to perform badly, cancelling the whole production may be too costly as also the output by other performers would be lost, meaning that the agent serves until the end of production, never to be offered another contract. In some settings, as in politics, the prohibition of renegotiation can be explained by voters differing in their policy preferences. With possible recalls in such a multiple-principals setting, there would be the risk of permanent campaigning, and most jurisdictions have decided against it (California being a notable exception). Early termination is common in some private sector contracts, whether for CEOs or sports coaches, and is often coupled with a severance pay. Allowing for renegotiation in our model would lead to precisely this result: the principal would offer an agent a severance pay to terminate a contract as soon as she finds out that the service quality is low.

Contract length is an ubiquitous dimension of contractual structure, which previous literature has typically neglected. We show that contract length plays a key role in principal-agent relationships. In the presence of moral hazard and adverse selection, contract design faces the double task of addressing screening and incentive problems. The optimal compensation structure then depends on whether contract length can be differentiated between initial and continuation contracts. As a first step, this paper has elaborated on some of the linkages and tradeoffs. Generalizations and modifications of our approach will certainly discover more.

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Appendix: Proofs

A.1 Proof of Proposition 1

A.1.1 Conditionally optimal duration of initial contracts

The proof of Proposition 1 builds on the following lemma, which characterizes the optimal duration of initial contracts, $S = S^*(T)$, conditional on the duration T of the continuation contract being given.

Lemma 1 *Suppose that T with $T \geq S_{min}$ is given. Then:*[10]

1. *If $\Gamma < 0$, then $S^*(T) = T$ for all T.*
2. *If $\Gamma > 0$, then:*

$$S^*(T) \begin{cases} = S_{min} & \text{if} \quad T \in (T_\ell, T_u) \\ \in [S_{min}, T] & \text{if} \quad T = T_\ell \quad \text{or} \quad T = T_u \\ = T & \text{if} \quad T \notin [T_\ell, T_u], \end{cases}$$

   where T_ℓ, T_u with $T_u > T_\ell$ are defined as

$$T_u, T_\ell = \frac{1}{2}\left(1 \pm \sqrt{\frac{\Gamma}{(1-\mu)p^2 v}}\right). \tag{11}$$

3. *If $\Gamma = 0$, then:*

$$S^*(T) \begin{cases} \in [S_{min}, 1/2] & \text{if} \quad T = 1/2 \\ = T & \text{if} \quad T \neq 1/2. \end{cases}$$

Proof of Lemma 1: Define

$$B(T) \quad := \quad (1-\mu)p^2 vT\,(1-T) - c\delta$$

to rewrite (8) as

$$\frac{\delta}{\mu} Z = pv + (1-S) \cdot \frac{B(T)}{D(S,T)}.$$

[10] Cases 2 and 3 could be collapsed into a single case with $\Gamma \geq 0$ since $T_\ell = T_u = 1/2$ for $\Gamma = 0$.

Then:

$$\frac{\delta}{\mu}\frac{\partial Z}{\partial S} = -\frac{B(T)}{D(S,T)^2} \cdot \left(D(S,T) + (1-S)\frac{\partial D}{\partial S} \right).$$

Calculate:

$$\begin{aligned} D(S,T) + (1-S)\frac{\partial D}{\partial S} &= (1-T) \cdot \Big[S(1-pT) + \mu p(1-S)T \\ &\quad +(1-S)\left((1-p(1-T)) - \mu pT\right) \Big] \\ &= (1-T)(1-p(1-T)), \end{aligned}$$

which is positive. Hence,

$$\begin{aligned} \frac{\partial Z}{\partial S} \leq 0 &\iff B(T) \geq 0 \\ &\iff T(1-T) \geq \frac{c\delta}{(1-\mu)p^2 v}. \end{aligned} \tag{12}$$

Observe that $0 \leq T(1-T) \leq 1/4$ for all $T \in [0,1]$. Hence:

1. $\Gamma < 0$ is equivalent to $\frac{c\delta}{(1-\mu)p^2 v} > 1/4$. Hence, from (12), $B(T) < 0$ for all T. Thus, $\frac{\partial Z}{\partial S} > 0$ for all (S,T). Hence, S should be chosen as large as possible, which means that $S = T$.

2. Suppose that $\Gamma > 0$. Then:

$$\begin{aligned} \frac{\partial Z}{\partial S} \substack{<\\=\\>} 0 &\iff T(1-T) \substack{>\\=\\<} \frac{c\delta}{(1-\mu)p^2 v} \\ &\iff \begin{cases} T \in (T_\ell, T_u) \\ T = T_\ell \quad \text{or} \quad T = T_u \\ T \notin [T_\ell, T_u], \end{cases} \end{aligned}$$

   where T_u and T_ℓ are, respectively, the smaller and the larger root of the quadratic equation $T(1-T) = \frac{c\delta}{(1-\mu)p^2 v}$, as given in (11). This implies that S should be as small as possible in the upper case, as large as possible in the lower case and can be arbitrarily chosen in the middle case:

$$S^* \begin{cases} = S_{min} & \text{if} \quad T \in (T_\ell, T_u) \\ \in [S_{min}, T] & \text{if} \quad T = T_\ell \quad \text{or} \quad T = T_u \\ = T & \text{if} \quad T \notin [T_\ell, T_u]. \end{cases}$$

3. Finally suppose that $\Gamma = 0$. Then:

$$\frac{\partial Z}{\partial S}\left\{\begin{matrix} = \\ > \end{matrix}\right\} 0 \quad \Longleftrightarrow \quad T \left\{\begin{matrix} = \\ \neq \end{matrix}\right\} \frac{1}{2}.$$

This implies that S can be arbitrarily chosen in the upper case and should be as large as possible in the lower case:

$$S^* \begin{cases} \in [S_{min}, 1/2] & \text{if} \quad T = 1/2 \\ = T & \text{if} \quad T \neq 1/2. \end{cases}$$

Observe that without loss in generality we can proceed with $S = T = 1/2$ whenever $\Gamma = 0$. •

A.1.2 Proof of Proposition 1 proper

Lemma 1 conveys that for $\Gamma < 0$ continuation and initial contract are of equal durations. Hence, the IC constraint (2) bites with equality. For $\Gamma > 0$, that constraint need not bind. We now show each item of Proposition 1 separately. For sake of easier reference, we start with the second item.

The case $\Gamma = 0$. Here, we can set $S^*(T) = T$ without loss in generality from Lemma 1. Solving $\Gamma = 0$ for v and replacing it in (5) and (6), we obtain:

$$Z(T,T)|_{\Gamma=0} = \frac{c\mu}{(1-\mu)p} \cdot \frac{4T - p(1-\mu)}{T(1-p(1-\mu)(1-T))} \tag{13}$$

and

$$V(T,T)|_{\Gamma=0} = \frac{c}{(1-\mu)p} \cdot \\ \cdot \frac{4(1-T)T(\mu + T(1-\mu)) - (1-\mu)(T + \mu p(1-T))}{(1-T)T(1-p(1-\mu)(1-T))}. \tag{14}$$

Differentiating (13) with respect to T gives:

$$\frac{\partial}{\partial T} Z(T,T)|_{\Gamma=0} = c\mu \cdot \frac{(1-2T)(1+2T-p(1-\mu))}{T^2(1-p(1-\mu)(1-T))^2}.$$

This has one positive zero at $T = 1/2$. It can be verified that the second-order derivative of Z is negative at $T = 1/2$. Hence, $Z(T, T)$ reaches its maximum at $T^* = 1/2$. From (13) and (14) we find

$$Z(1/2, 1/2)|_{\Gamma=0} = V(1/2, 1/2)|_{\Gamma=0} = \frac{4c\mu}{(1-\mu)p}, \tag{15}$$

which is strictly positive. Hence, participation and IC constraints are both satisfied. This completes the proof.

The case $\Gamma > 0$. From (5),

$$\frac{\partial Z(S,T)}{\partial T} = \frac{\mu(1-S)}{\delta} \cdot \frac{N(S,T)}{D^2(S,T)}, \tag{16}$$

where

$$\begin{aligned} N(S,T) \quad := \quad & -c\delta\left[S + \mu p(1-S) - 2p(1-T)(S + \mu(1-S))\right] \\ & +(1-\mu)(1-p)p^2 S(1-T)^2 v. \end{aligned}$$

Here, $D^2(S,T)$ is positive for all (S,T). Hence, the sign of $\frac{\partial Z(S,T)}{\partial T}$ is determined by the sign of $N(S,T)$. Observe that, for given S, every *admissible* critical point of $Z(.,T)$ is a local maximum. I.e., if $T^* > 0$ solves $\frac{\partial Z(S,T)}{\partial T} = 0$ for some S, then $\frac{\partial^2 Z(S,T^*)}{\partial T^2} < 0$.[11]

We will now show that the initial contract optimally is as short as possible: $S = S_{min}$. From Lemma 1, item 2, this will hold whenever $T \in (T_\ell, T_u)$, as defined in (11). To see that this holds, we evaluate $N(S,T)$ at T_ℓ and T_u. We can write $T_u, T_\ell = 1/2(1 \pm r)$ where $r := \sqrt{1 - \frac{4c\delta}{(1-\mu)p^2 v}}$ is well-defined and has $r \in (0,1)$. Calculate that

$$N(S,T_\ell) \quad = \quad -c\delta[S + \mu p(1-S) - p(1+r)(S + \mu(1-S))]$$

[11]If T^* solves $N(S,T^*) = 0$ then:

$$\frac{\partial^2 Z(S,T^*)}{\partial T^2} = \frac{\mu(1-S)}{\delta} \cdot \frac{N_T(S,T^*)}{D^2(S,T^*)}.$$

Since $D^2 > 0$ and $N_T(S,T^*) = -2[pc\delta(S + \mu(1-S)) + (1-T^*)(1-\mu)(1-p)p^2 S v] < 0$, this expression is negative at T^*.

$$
\begin{aligned}
& \quad +\frac{1}{4}(1-\mu)(1-p)p^2S(1+r)^2v \\
&= -c\delta[(1-p)S - pr(S+\mu(1-S))] + \frac{1}{4}(1-\mu)(1-p)p^2S(1+r)^2v \\
&> -c\delta(1-p)S + \frac{1}{4}(1-\mu)(1-p)p^2S(1+r)^2v \\
&= (1-p)S\left(-c\delta + \frac{1}{4}(1-\mu)p^2v(1+r)^2\right) \\
&= \frac{1}{4}(1-\mu)p^2(1-p)Sv\left(-(1-r^2)+(1+r)^2\right) \\
&= \frac{1}{2}(1-\mu)p^2(1-p)Sv(r+r^2) > 0,
\end{aligned}
$$

where we replaced $c\delta = \frac{1}{4}(1-r^2)(1-\mu)p^2v$. Likewise,

$$
\begin{aligned}
N(S,T_u) &= -c\delta[S+\mu p(1-S) - p(1-r)(S+\mu(1-S))] \\
& \quad +\frac{1}{4}(1-\mu)(1-p)p^2S(1-r)^2v \\
&= -c\delta[(1-p)S + pr(S+\mu(1-S))] + \frac{1}{4}(1-\mu)(1-p)p^2S(1-r)^2v \\
&< (1-p)S\left(-c\delta + \frac{1}{4}(1-\mu)p^2(1-r)^2v\right) \\
&= (1-p)S\left(-\frac{1}{4}(1-\mu)p^2v(1-r^2) + \frac{1}{4}(1-\mu)p^2v(1-r)^2\right) \\
&= \frac{1}{2}(1-\mu)p^2(1-p)Sv(r^2-r) < 0,
\end{aligned}
$$

since $0 < r < 1$. In summary,

$$
\frac{\partial Z(S,T_\ell)}{\partial T} > 0 > \frac{\partial Z(S,T_u)}{\partial T} \quad \text{for all}\, S.
$$

Hence, for any S, the optimal duration of the continuation contract is in (T_ℓ, T_u). From item 2 in Lemma 1, the initial contract has minimum duration $S = S_{min}$.

The optimal contract duration can, thus, be derived from maximizing $Z(S_{min}, T)$ by choice of $T \geq S_{min}$, obeying the principal's IC constraint (7) with $S = S_{min}$. Recall from (16) that $Z(S,T)$ increases in T whenever $N(S,T)$ is positive. Now verify that

$$
N(S, 1/2) = (1-p)S\Gamma/4,
$$

which is positive for any S (and, in particular for S_{min} if and only if $\Gamma > 0$ – which we currently assume. Hence, at $(S, T) = (S_{min}, 1/2)$, payoffs Z can be increased by increasing T. This implies that $T^* > 1/2$.

Above we showed that for $\Gamma = 0$ both Z and V are strictly positive in the optimum (see (15)). By the Envelope Theorem, the maximum values of Z and V increase in v, p and decrease in c and δ. Hence, $V(S_{min}, T^*)$ and $Z(S_{min}, T^*)$ are strictly positive, too, and the principal's participation and IC constraints are satisfied. This completes the proof for the case $\Gamma > 0$.

The case $\Gamma < 0$. This case follows directly from Lemma 1. The optimal duration of initial and continuation contract as well as the principal's participation constraint will be characterized in Proposition 3 below.

A.2 Proof of Proposition 2

As $v > \hat{v}$ is equivalent to $\Gamma > 0$, we have $S^* = S_{min}$ from item 3 in Proposition 1. The optimal duration of the continuation contract is, thus, given by

$$T^m := \arg \max_{T \geq S_{min}} Z(S_{min}, T),$$

obeying the principal's IC constraint (7) with $S = S_{min}$. The first-order condition for the corresponding maximization requires that $N(S_{min}, T) = 0$, which gives two solutions:

$$T_- = 1 - \frac{R - A}{(1-\mu)(1-p)pS_{min}v} \quad \text{and} \quad T_+ = 1 + \frac{R - A}{(1-\mu)(1-p)pS_{min}v}, \tag{17}$$

where

$$\begin{aligned} A &= c\delta(S_{min} + (1 - S_{min})\mu), \\ R &= \sqrt{A^2 + c\delta(1-\mu)(1-p)S_{min}(S_{min} + (1 - S_{min})\mu p)v}. \end{aligned}$$

Since $R > A$, T_+ exceeds one and, thus, is not an admissible solution. By contrast, $T^m := T_-$ is smaller than one. Moreover, as shown in item 3 of Proposition 1, $T^m > 1/2$. For the monotonicity of T^m in v, recall that T^m is implicitly defined by $\partial Z/\partial T = 0$ (see 16). Hence,

$$\frac{\partial T^m}{\partial v} = -\frac{\partial^2 Z(S_{min}, T^m)/(\partial T \partial v)}{\partial^2 Z(S_{min}, T^m)/\partial T^2}.$$

The denominator is negative by the SOC. From (16), the numerator is equal in sign to $\frac{\partial N(S_{min},T^m)}{\partial v}$, since $D(S,T)$ does not depend on v. However, N is increasing in v, implying that T^m increases in v.

A.3 Proof of Proposition 3

Define

$$T^s := \quad \arg\max_{T \geq S_{min}} Z(T,T), \tag{18}$$

$$T^0 := \quad \max\{T \geq S_{min} \,|\, V(T,T) = 0\}. \tag{19}$$

Here, T^s and T^0 denote respectively, the *unconstrained* maximizer of the principal's payoff function and the longest duration T such that the principal's IC constraint is not violated, each defined for the case that initial and continuation contracts are of equal duration ($S = T$).

Using the definitions of T^s and T^0, Proposition 3 can be re-phrased more technically as follows:

Proposition 6 *There exist values* $0 < \underline{v} < v_0 < \hat{v}$ *such that:*

1. *If* $v < \underline{v}$*, no contract will be offered.*
2. *If* $\underline{v} \leq v \leq v_0$*, initial and continuation contracts both have an identical duration* $T^* = T^0$*. Here,* T^0 *increases in* v *with* $T^0 < 1/2$ *at* $v = \underline{v}$ *and* $T^0 > 1/2$ *at* $v = v_0$*.*
3. *If* $v_0 < v < \hat{v}$*, initial and continuation contracts both have optimal duration* $T^* = T^s$*. Here,* T^s *strictly decreases in* v *and is larger than* $1/2$*.*

Suppose that $v < \hat{v}$ or, equivalently, $\Gamma < 0$. Thus, $S^* = T^*$ from item 2 in Proposition 1). Using this, we rewrite the principal's total payoff as

$$\begin{aligned} \tilde{Z}(T) := Z(T,T) &= \frac{\mu}{\delta}\left[pv + \frac{(1-\mu)p^2T(1-T)v - c\delta}{T\,(1-p(1-\mu)(1-T))}\right] \\ &= \frac{\mu}{\delta} \cdot \frac{pTv - c\delta}{T(1-p(1-\mu)(1-T))}. \end{aligned} \tag{20}$$

We are looking for the solution of the constrained optimization problem:

$$\max_{T \geq S_{min}} \tilde{Z}(T) \quad \text{s.t.} \quad \tilde{V}(T) \geq 0, \tag{21}$$

where

$$\begin{aligned} \tilde{V}(T) &:= V(T,T) \qquad (22) \\ &= \frac{1}{p\delta D(T,T)} \left[-c\delta(\mu p + (1-\mu p)T) + vp^2(1-T)T(\mu + (1-\mu)T) \right] \end{aligned}$$

denotes the value from the continuation contract, provided that $S = T$. Obviously, we can re-write (18) and (19) as

$$T^s := \quad \arg \max_{T \in (S_{min},1)} \tilde{Z}(T) \tag{23}$$

$$T^0 := \quad \max\{T \in (S_{min}, 1) | \tilde{V}(T) = 0\}. \tag{24}$$

First recall from the proof of Proposition 1 that, at $v = \hat{v}$, the optimal solution to (21) has $T^* = 1/2$ with $V(1/2) > 0$. Hence, the IC constraint in (21) does not bind at $v = \hat{v}$. By continuity, for values of v smaller but sufficiently close to $\hat{v}$, the solution of (21) is given by T^s.

Calculate that:

$$\begin{aligned} \tilde{Z}_T(T) &:= \frac{\partial \tilde{Z}(T)}{\partial T} \\ &= \frac{\mu}{\delta} \cdot \frac{c\delta(1-(1-\mu)p(1-2T)) - (1-\mu)p^2T^2v}{T^2(1-p(1-\mu)(1-T))^2}. \end{aligned} \tag{25}$$

The sign and the zeros of $\tilde{Z}_T$ are given by the sign and zeros of its numerator, which is quadratic in T. First observe that at $T = 1/2$ the numerator becomes $c\delta - (1-\mu)p^2v/4$, which is positive for $v < \hat{v}$. Hence, $\tilde{Z}_T(1/2) > 0$.

The two zeros of $\tilde{Z}_T(T)$ are

$$T_{+,-} = \frac{(1-\mu)c\delta \pm \sqrt{c\delta(1-\mu)(v(1-p(1-\mu)) + c\delta(1-\mu))}}{(1-\mu)pv}.$$

Here, the root is larger than $(1-\mu)c\delta$. Hence, T_- is negative and, thus, inadmissible for all parameters. However, $T_+ > 1/2$ if and only if $v < \hat{v}$, as assumed. Since $\tilde{Z}_T(1/2) > 0$, the maximum of $\tilde{Z}(T)$ is indeed achieved at T_+. Hence, as long as the constraint $\tilde{V}(T_+) \geq 0$ does not bite,

$$
\begin{aligned}
T^* = T^s(v) = T_+ &= \frac{c\delta}{pv} + \frac{\sqrt{c\delta(1-\mu)(v(1-p(1-\mu)) + c\delta(1-\mu))}}{(1-\mu)pv} \\
&= \frac{c\delta}{pv}\left(1 + \sqrt{1 + \frac{v}{c\delta}\left(\frac{1}{1-\mu} - p\right)}\right). \qquad (26)
\end{aligned}
$$

A more valuable project requires shorter contracts: $\frac{\partial T^s}{\partial v} < 0$. Moreover, T^s approaches $1/2$ when v comes close to $\hat{v}$ from above.

Provided that $\tilde{V}(T^s(v)) \geq 0$, we have $T^* = T^s(v)$. Recalling that $\tilde{V}(T^s(v)) > 0$ if v is close to $\hat{v}$, the IC constraint is satisfied in these cases. However, when v decreases, the IC constraint $\tilde{V}(T) \geq 0$ eventually will bite (observe that both $\tilde{V}$ and $\tilde{Z}$ are linear in v).

The smallest flow value v such that $\tilde{V}(T^s(v))$ is non-negative can be calculated as[12]

$$
\begin{aligned}
v_0 = &\frac{c\delta}{2(1-\mu)\mu^2p^2} \cdot \qquad (27) \\
&\left(1 - p(1 + 4\mu^3 - 6\mu^2 + \mu) - (1 - p + \mu(p-2))\sqrt{1 + 4\mu - 4\mu^2}\right);
\end{aligned}
$$

the attending contract duration $T^s(v_0)$ can be calculated from (26). Note that $v_0 < \hat{v}$ for all parameters.[13] As $T^s(\hat{v}) = 1/2$ from Proposition 1 and $\partial T^s/\partial v < 0$, we obtain $T^s(v_0) > 1/2$.

As $\tilde{V}(T)$ strictly increases in v while $T^s(v)$ decreases in v, the IC constraint $\tilde{V}(T) \geq 0$ is binding at the optimal T for all $v \leq v_0$. Hence, the optimal solution to (21) is given by the largest[14] T such that the IC con-

[12]These calculations were run with the help of Mathematica software. The corresponding notebook is available on request.

[13]Verify that:

$$
\hat{v} - v_0 = \frac{c\delta\left(2\mu^2(4 + 2\mu p - 3p) + (1 - p + \mu(p-2))\sqrt{1 + 4\mu - 4\mu^2} + \mu p + p - 1\right)}{2(1-\mu)\mu^2p^2}
$$

The numerator is zero at $(p, \mu) = (0, 0)$. Moreover, it is strictly increasing both in p and in μ on $(p, \mu) \in (0, 1)^2$. (There is a non-differentiability at all (p, μ) with $\mu = (1-p)/(2-p)$. This does not affect monotonicity properties, though.) Hence, $v_0 < \hat{v}$ for all $(p, \mu) \in (0, 1)^2$.

[14]Observe that $\tilde{V}(T) \to -\infty$ for $T \to 1$. Hence, we are indeed looking for the largest T that satisfies $\tilde{V}(T) \geq 0$.

straint just holds: for $v < v_0$,

$$T^* = T^0,$$

as defined in (24). We will check the participation constraint $\tilde{V}^s(T^0) \geq 0$ below.

Suppose that $v \leq v_0$. Let us study the binding IC constraint $\tilde{V}(T) = 0$ in more detail. Using (1), (3), (4), $T = S$, and $w^s = 0$ we obtain that $\tilde{V}(T) = 0$ is equivalent to

$$G(T, v) := \left(pvT - \frac{\delta c}{p(1-T)} \right) + \mu p(1-p)v \frac{T(1-T)}{1-(1-\mu p)(1-T)} = 0.$$

From (4) with $V = 0$, the second term in this sum equals $\delta(1-p)(1-T)\tilde{Z}(T)$, which must be non-negative. Hence, $G = 0$ implies

$$pvT - \frac{\delta c}{p(1-T)} < 0. \tag{28}$$

Replace $w(T)$ from (1) this means that

$$T(pv - w(T)) < 0$$

— the principal makes a loss from keeping a successful, high-quality agent. Clearly, $T^0 = T^0(v)$ is the (largest) solution to $G(T^0(v), v) = 0$. By the Implicit Function Theorem,

$$\frac{\partial T^0}{\partial v} = -\frac{G_v(T^0, v)}{G_T(T^0, v)}$$

where

$$\begin{aligned} G_v(T, v) &= pT + \mu p(1-p) \frac{T(1-T)}{1-(1-\mu p)(1-T)} > 0, \\ G_T(T, v) &= \left(pv - \frac{\delta c}{p(1-T)^2} \right) + \mu p(1-p)v \frac{1-2T-(1-\mu p)(1-T)^2}{(1-(1-\mu p)(1-T))^2}. \end{aligned}$$

At and closely below $v = v_0$, we have $T^0 > 1/2$; this follows by continuity since $T^0 = T^s > 1/2$ at $v = v_0$. Hence, the second term in G_T is negative. The first term is negative, too. This follows from (28) and $T > 1/2$:

$$pv - \frac{\delta c}{p(1-T)^2} < pv - \frac{\delta c}{pT(1-T)} < 0.$$

In summary, $\frac{\partial T^0}{\partial v} > 0$ for v close enough to v_0.

Next observe from (3) and (4) that the principal's participation constraint $\tilde{Z} \geq 0$ always holds as long as the IC constraint $\tilde{V} \geq 0$ is satisfied. Hence, the smallest flow value $\underline{v}$ such that a contract will be offered satisfies[15]

$$\tilde{V}(T^0(\underline{v})) = \tilde{V}_T(T^0(\underline{v})) = 0. \tag{29}$$

Verify that $\tilde{V}(T) = \frac{G}{\delta(1-p(1-T))}$.[16] Hence, if $\tilde{V} = G = 0$, then

$$\tilde{V}_T = \frac{1}{\delta(1-p(1-T))^2}\left((1-p(1-T))G_T - pG\right) = \frac{G_T}{\delta(1-p(1-T))},$$

such that $\tilde{V} = \tilde{V}_T = 0$ necessitates $G_T = 0$. Consequently, for $v \leq v_0$,

- the optimal contract duration is $T^* = T^0(v)$;
- a contract will be offered if and only if $v \geq \underline{v}$, as implicitly defined in (29);
- at $v = \underline{v}$ we have $T^* < 1/2$;[17]
- T^* is increasing in v on the interval $(\underline{v}, v_0)$;[18]

[15]Geometrically, given v, the function $\tilde{V}(T)$ is inversely u-shaped in a (T, v)-diagram, cutting the axis $V = 0$ at most twice, the first time from below. Lowering v shifts the $\tilde{V}$-curve downwards. We are looking for the smallest v such that there exists some T with $\tilde{V}(T) \geq 0$; this T-value must then be the maximizer of $\tilde{V}$ and, thus, satisfies $\tilde{V}_T(T) = 0$.

Moreover, $\underline{v} < v_0$: At $v = v_0$ we have $\tilde{V}(T^s(v_0)) = 0$ with $T^s(v_0) > 1/2$ (see above). Hence, $\tilde{V}_T(T^s(v_0)) < 0$, implying that for v_0 there exist $T < T^s(v_0)$ with $\tilde{V}(T)$ strictly positive. Since $\tilde{V}$ strictly increases in v, we must therefore have $\underline{v} < v_0$.

[16]Expression (3) can be rewritten as

$$V = \frac{1}{\delta}(pv - w)\frac{T}{1-p(1-T)} + \frac{(1-p)(1-T)}{1-p(1-T)}Z;$$

the same relationship also holds between $\tilde{V}$ and $\tilde{Z}$. Multiplying by $\delta(1-p(1-T))$ gives:

$$\delta(1-p(1-T))\tilde{V} = (pvT - \frac{\delta c}{p(1-T)}) + \delta(1-p)(1-T)\tilde{Z} = G.$$

[17]This follows from $G_T(T^*(\underline{v}), \underline{v}) = 0$, which necessitates $T < 1/2$. (Recall that $G_T(T) < 0$ for $T \geq 1/2$. Whenever $G_T = 0$, we have $T < 1/2$.)

[18]Above we showed that $G_T < 0$ initially, implying that T^* increases in v. Now the largest admissible value of v has $G_T(T^*(\underline{v}), \underline{v}) = 0$. So G_T will never be positive; hence, T^* can never decrease in v on $(\underline{v}, v_0)$.

- around $\underline{v}$, T^* is locally convex in v.[19]

This establishes the three items in Proposition 3.

A.4 Proof of Proposition 4

The principal chooses (S, T) such as to maximize (4), obeying $w^s \geq 0$, (1), (10), and $Z \geq 0$.

Let us first suppose that $w^s > 0$. As $\frac{\partial Z}{\partial w^s} < 0$, the constraint (10) always bites. Thus,

$$w^s + pv(1 - \mu) = w. \tag{30}$$

Plugging this into (5) yields:

$$Z(S, T) = \frac{pv}{\delta} - \frac{c}{pT(1 - T)}. \tag{31}$$

This is independent of S and will be maximal if $T = 1/2$. The attending wage levels for the initial and the continuation contracts then are

$$w = 4\delta c/p \quad \text{and} \quad w^s = 4\delta c/p - pv(1 - \mu).$$

As w^s has to be non-negative, this is only admissible if $v \leq 4\delta c/[p^2(1-\mu)] = \hat{v}$ or $\Gamma \leq 0$.

The principal's payoff at $T = 1/2$ equals $Z = \frac{pv}{\delta} - \frac{4c}{p}$, which is positive only if $v \geq 4\delta c/p^2$ or $\Gamma \geq -4\mu\delta c$. Below that value of Γ, no contract will be offered, proving the first item of the claim.

Since $Z(S, T)$ in (31) does not vary with S in this case, we can always satisfy the agent's IC constraint $S \leq T$ by choosing S arbitrarily from $[S_{min}, 1/2]$.

Thus, contract structures with $T^* = 1/2$ and $S \in [S_{min}, 1/2]$ satisfy all Kuhn-Tucker conditions for the constrained optimization if $\Gamma \leq 0$. Such contract structures are, however, not admissible if $\Gamma > 0$, since then w^s would be negative. This proves the second item in Proposition 4.

Now suppose that $\Gamma > 0$. By the argument above, the initial contract then must not pay anything: $w^s = 0$. We are now back in the scenario of

[19] At $v = \underline{v}$ we have $G_T(T^*(\underline{v}), \underline{v}) = 0$; hence $T^*(v)$ is (locally) flat (only the left-hand derivative is defined, however). When v increases, T^* increases. Hence, it is locally convex in v.

the first item in Proposition 1. We have already checked that the principal's participation constraint $Z > 0$ holds for $\Gamma \geq 0$. It remains to be shown that the constraint (10) also holds at (17). With $w^s = 0$ and (1), the constraint requires that

$$T(1-T) \geq \frac{c\delta}{p^2 v(1-\mu)},$$

which is exactly condition (12) from the proof of Lemma 1. Since $\Gamma > 0$, it is equivalent to $T \in (T_u, T_\ell)$, as defined in (11). Recall from the proof of item 1 in Proposition 1 that the optimal contract satisfies this. Hence, (10) holds.

For the wages $w(T^m)$ paid in the continuation contract, we obtain:

$$\frac{\partial w(T^m)}{\partial v} = w'^m(T^m) \cdot \frac{\partial T^m}{\partial v}.$$

Recalling that $T > 1/2$ we get $w'(T) > 0$, rendering the entire expression positive. This completes the proof of the third item in Proposition 4.

A.5 Proof of Proposition 5

Plugging $w^s = w = w(T)$ and $S = T$ into (3) and (4) leads to

$$\tilde{Z}(T) = \frac{H(T) - w(T)}{\delta},$$

where

$$H(T) := \frac{p\mu v}{1 - p(1-T)(1-\mu)}$$

is positive and decreases in T. As $\tilde{Z}(T)$ is negative both for small and large values of T (since $w(T)$ grows beyond all bounds), the maximizer of $\tilde{Z}$ is in the interior of $(0, 1)$ and satisfies the first-order condition $\frac{\partial \tilde{Z}(T)}{\partial T} = H'(T) - w'(T) = 0$. Since $H'(T) < 0$ this can only hold if $w'(T) < 0$ or, equivalently, if $T < 1/2$. Since $H'(T)$ decreases in v while $w'(T)$ does not depend on v, the comparative statics $\partial T^*/\partial v < 0$ follow directly the Implicit Function Theorem applied to the first-order condition.